Mathematics for the Ears

An Explanation of the Hidden Mathematical Structures in Post-Tonal Music

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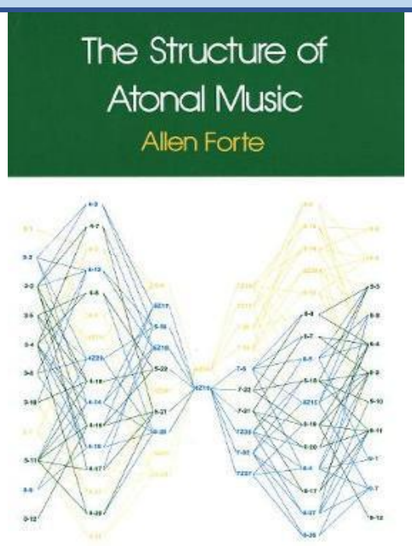
Introduction

During the 20th century, music theorists began to be influenced by mathematical concepts when analysing music. These basic mathematical concepts and tools they used quickly formed two related but distinct fields of music analysis, Musical Set Theory, and Transformation Theory. These two fields do not contain much in the way of advanced mathematics; most of it should be understandable to someone who has taken a year or two of an undergraduate mathematics course. However, the underlying mathematics used in these fields is often brushed over in textbooks or papers in favour of the actual applications in analysis.

This creates a barrier between musicians working in these fields, and mathematicians interested in them. Mathematicians in general are interested in applying maths to other fields, and I believe that more progress and more new ideas can be brought forward by mathematicians, who may not be experts in music but still understand the underlying systems and relations due to the mathematical nature of these fields



"Music is the hidden arithmetical exercise of a mind unconscious that it is calculating."

— Gottfried Wilhelm Leibniz

Goals

This project had two main goals:

- To create a handbook aimed at mathematicians or mathematics students to explain the mathematics of Musical Set Theory and Transformation Theory.
- To determine levels of interest among mathematics and music students to learn more about how each others fields can apply to their own fields

Methodology

The Handbook was written over the course of the two summers. Concepts discussed in music theory textbooks without any discussion of the mathematics being used were rewritten to explicitly show the mathematics, and any proofs that should have been given were proven. Proofs had to be done by myself and my supervisor, as proofs for any theorems or lemmas in this area are exceedingly difficult to find.

Two surveys were made, one each for music students and maths students. The questions correspond with each other, but they both contain questions specific to the fields of the recipients. The survey for mathematics students was disseminated by the TCD mathematics society, and the survey for music students was sent to students in each year group who shared it among their classmates by means of online group chats. A space for free text responses was also left at the end of each survey.

Results

Material for the handbook was gathered during the first research period, and it was edited during the second research period. While not completely comprehensive, it covers the basics of both fields, and many of the most important concepts. The handbook should be understandable to anybody who has taken an introductory group theory module in University, as well as done a small amount of work in other areas such as metric spaces. Somebody who has completed second year mathematics in TCD should be able to understand everything in it.

The surveys had a higher than expected response rate, with 40 responses from mathematics students and 32 from music students. The most interesting results were that the majority of both groups believed that mathematics could be used in the composition of good music, and that both groups were generally interested in learning about the other's field, that is, musicians were interested in learning more about the mathematics behind the areas of music analysis in question, and mathematicians were interested in learning about how music can be related to mathematics.

Responses

Below are two free text responses, which are broadly representative of the opinions of the respondents.

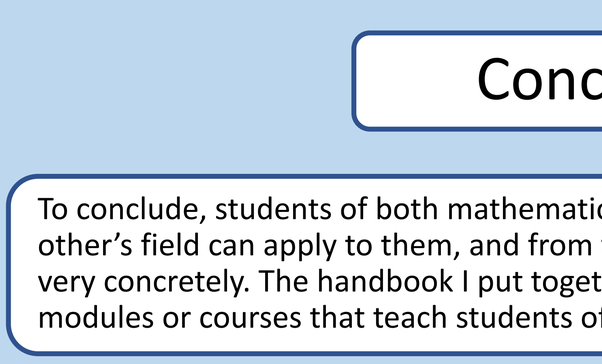
"Music has chordal structures (eg 1-4-5-1) and other rigorous mathematical backbone inherent to it. Therefore, using a mathematical viewpoint there is something to be learned from music using this lens. Likewise creating music through maths is also possible if not encouraged."

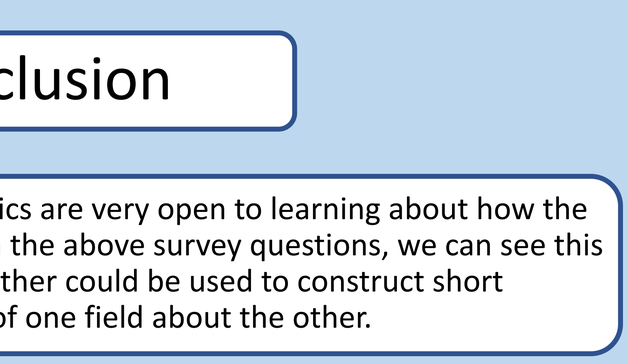
Mathematics Student.

"I think maths and music are so closely related, and learning more about these two subjects would be useful to skills needed in careers"

Music Student

Survey Results





Conclusion

To conclude, students of both mathematics are very open to learning about how the other's field can apply to them, and from the above survey questions, we can see this very concretely. The handbook I put together could be used to construct short modules or courses that teach students of one field about the other.

Resources

Handbook pdf — https://bit.ly/2SMVJlp

Survey Results — https://bit.ly/34NMdEd

References

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- Further References available on request