

Dissipative quantum phase transitions in non-Markovian systems

Riccardo Bonzano, supervised by Dr Brendon Lovett
School of Physics and Astronomy, University of St Andrews



1. Introduction

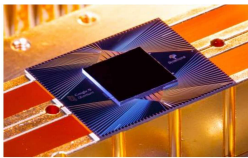
• Dissipative quantum phase transitions

A quantum phase transition is a phase transition which occurs at zero temperature in response to the variation of some non-thermal parameter [1], [2]. Let us consider an open system (i.e. it interacts with an environment or bath) which experiences dissipation. The dissipative coupling to the bath has a considerable impact on the system dynamics and can trigger interesting phase transitions: in this case, we talk about dissipative quantum phase transitions (DQPT) [3].

• The spin-boson model

The spin-boson model (SBM) describes the effects of dissipation on a system allowed to assume only two different states: spin up and spin down (Fig.2). We consider an equal superposition of the two spin states (i.e. the unbiased case). The system-bath interaction is quantified by the coupling strength α , a non-thermal parameter. Depending on the bath characteristics, the SBM can be sub-Ohmic, Ohmic and super-Ohmic [4]. A deeper understanding of this model as a qubit is needed for the practical realization of quantum computers [5].

Fig.1: Photograph of Google's Sycamore 54 qubit quantum processor. Adapted from [6].



• The localization transition

At a critical value of the coupling strength $\alpha = \alpha_c$, the system wavefunction is collapsed and the superposition destroyed: the system is localized into either one of the two spin states [4]. Intuitively, any quantum mechanical behaviour by the system is destroyed when the environment "looks" at it [7]. At low values of α , the system remains in the superposition: it is in the delocalized phase. This phase transition is a DQPT.

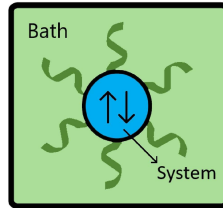


Fig.2: Schematic representation of the SBM. The system is embedded in the bath and can assume the spin up and down states. The darker green curly lines represent dissipation. Adapted from [8].

• Non-Markovian systems & the TEMPO algorithm

The state of a Markovian system is determined solely by its direct previous state [9]. Contrarily, the memory-effect is non-negligible in non-Markovian systems [10]; this makes their modelling challenging, as computational memory requirements increase very rapidly [8]. The localization transition in the SBM is non-Markovian. The TEMPO (time evolving matrix product operators) algorithm, developed by Dr Lovett and his group, is a computationally efficient, general and numerically exact method to model these kinds of processes [8].

2. Aims and Methods

• **Summer 2019:** to cover the basics of the background theory of the SBM and its Ohmic localization transition. To verify the consistency of a version of TEMPO, coded in Python by PhD student Gerald Fux, with the results of [8], and to reproduce interesting graphs (Fig.1,2,3 in [8]).

• **Summer 2020:** to write a review article on the sub-Ohmic SBM and the current stage of research in the area. To reproduce, by using TEMPO, interesting sub-Ohmic SBM dynamics and investigate a fascinating phase transition graph by Thorwart et al (Fig.1,5 respectively in [11]).

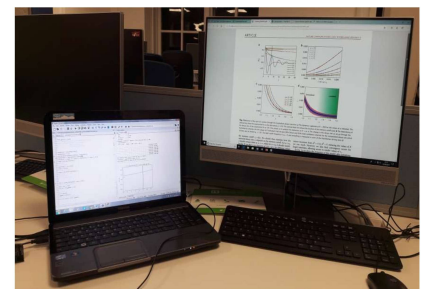


Fig.3: My workstation at the university library on my 1st year of research. Own photograph.

3. Major results

The graphs on this page model interesting SBM physics. They display the expectation value of the spin $\langle S_z \rangle$ as a function of time. Initially, at $t = 0$, the system is prepared in the spin up state: $\langle S_z \rangle = 1/2$. When time starts flowing, the system-bath interaction begins. For $\alpha = 0$, the system would always be isolated from the bath, and we would see an undamped oscillation between $+1/2$ and $-1/2$, as expected from an unbiased superposition state. The black vertical line is the memory cut-off, which indicates for how long the algorithm retains memory.

• Summer 2019: the Ohmic localization transition

In the graph below (Fig.4), we look at the Ohmic regime at $T = 0$: the dynamics of the system is plotted for different values of α . For $\alpha < \alpha_c$, $\langle S_z \rangle$ decays to zero, either through underdamped ($\alpha = 0.1, 0.2$) or critically damped ($\alpha = 0.7, 1.0, 1.2$) oscillations. The dashed lines are the exponential decay fits. Clearly, for $\alpha < \alpha_c$ the effect of the bath is not strong enough to destroy the superposition, and in the long-time limit there is an equal probability for the system to be found in the spin up or down state. For $\alpha > \alpha_c$, the system remains at $+1/2$ ($\alpha = 1.5$): the bath properly "looks" at the system and causes its localization in the state it is initially prepared. The localization transition occurs at $\alpha_c = 1.25$.

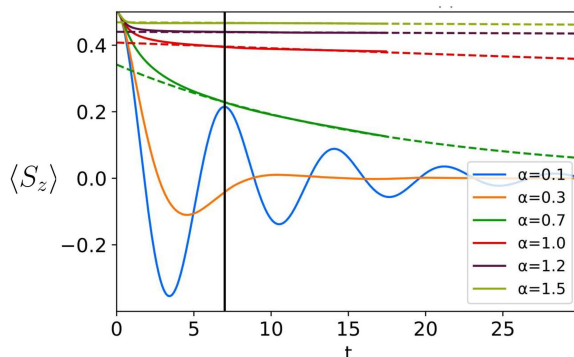


Fig.4. Reproduction of Fig.1 in [8]. Own figure.

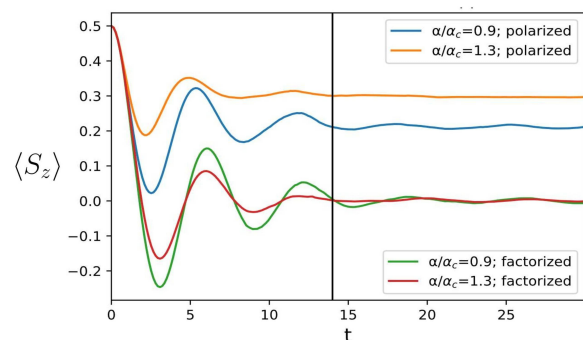


Fig.5. Reproduction of Fig.1 in [11]. Own figure.

• Summer 2020: a biased delocalization in the sub-Ohmic regime

In the graph above (Fig.5), we look at the sub-Ohmic regime close to $T = 0$: the dynamics differs based on α ($\alpha_c = 0.022$) and on the initial preparation of the system. The values of α displayed place the system in the delocalized phase. In the factorized case, the system at $t = 0$ is isolated from the bath and in equilibrium only with itself (this is also the initial preparation in Fig.4); the underdamped oscillations occur around zero as expected from an unbiased superposition state. In the polarized case, on the other hand, the system is initially in equilibrium with the bath and, interestingly, the oscillations occur around a finite value. This is because it takes time for the bath to respond to changes in the system; meanwhile, the former pulls the latter back to the initial condition of equilibrium. In other words, the bath "wants" the system to stay where it is, which causes a biased delocalization.

4. Acknowledgements

Firstly, I would like to thank my supervisor, Dr Brendon Lovett, who offered me the invaluable opportunity to gain insight into such a fascinating and cutting-edge area of physics and have a glimpse of what is like to do research, and who always supported me throughout the two years of my project. I would also like to thank Gerald Fux, who helped me countless times with the code and gave me precious advice whenever I got stuck. Finally, I would like to thank Lord Laidlaw, who made the scholarship possible, and to all the Laidlaw Team who made this experience profoundly constructive for my personal and academic future.

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