Formation of Einstein – Dirac solitons: a phase transition in the early universe

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Introduction

- **Einstein – Dirac system**

The Einstein – Dirac (ED) system of equations is a semi-classical model of quantum gravity which couples Einstein field equations with the Dirac equation [1]. We modify this system by considering a non-zero cosmological constant.

$$\sqrt{A}\alpha' = \frac{N\sigma}{2r}\alpha - (\omega T + m)\beta \tag{1}$$

$$\sqrt{A}\beta' = -\frac{N\sigma}{2r}\beta + (\omega T - m)\alpha. \tag{2}$$

$$\frac{1}{r^2}(rA' - 1 + A + \Lambda r^2) = -8\pi N\omega T^2\frac{(\alpha^2+\beta^2)}{r^2} = -8\pi\rho \tag{3}$$

$$\frac{1}{r^2}\left(2rA\frac{T'}{T} - A + 1 - \Lambda r^2\right) = \frac{-8\pi NT}{r^2}\left(\omega T(\alpha^2+\beta^2) - \frac{N\sigma}{r}\alpha\beta - m(\alpha^2-\beta^2)\right) = -8\pi P_r \tag{4}$$

Motivation

- **A phase transition in the early universe**

The project aims to investigate whether Einstein-Dirac solitons (EDS) occur when fermions are embedded in a uniform matter background. The phase transition that drives fermionic condensation in solitonic structures mimics particle generation in the early universe (i.e., during the Quark Epoch) [2].

Results: 1) EDS with a non-zero cosmological constant

We solve the ED system numerically by implementing the procedure in [1]. We find EDS in the presence of both positive and negative non-zero cosmological constant Λ. An example is shown in Fig. $1a$ and $1b$.

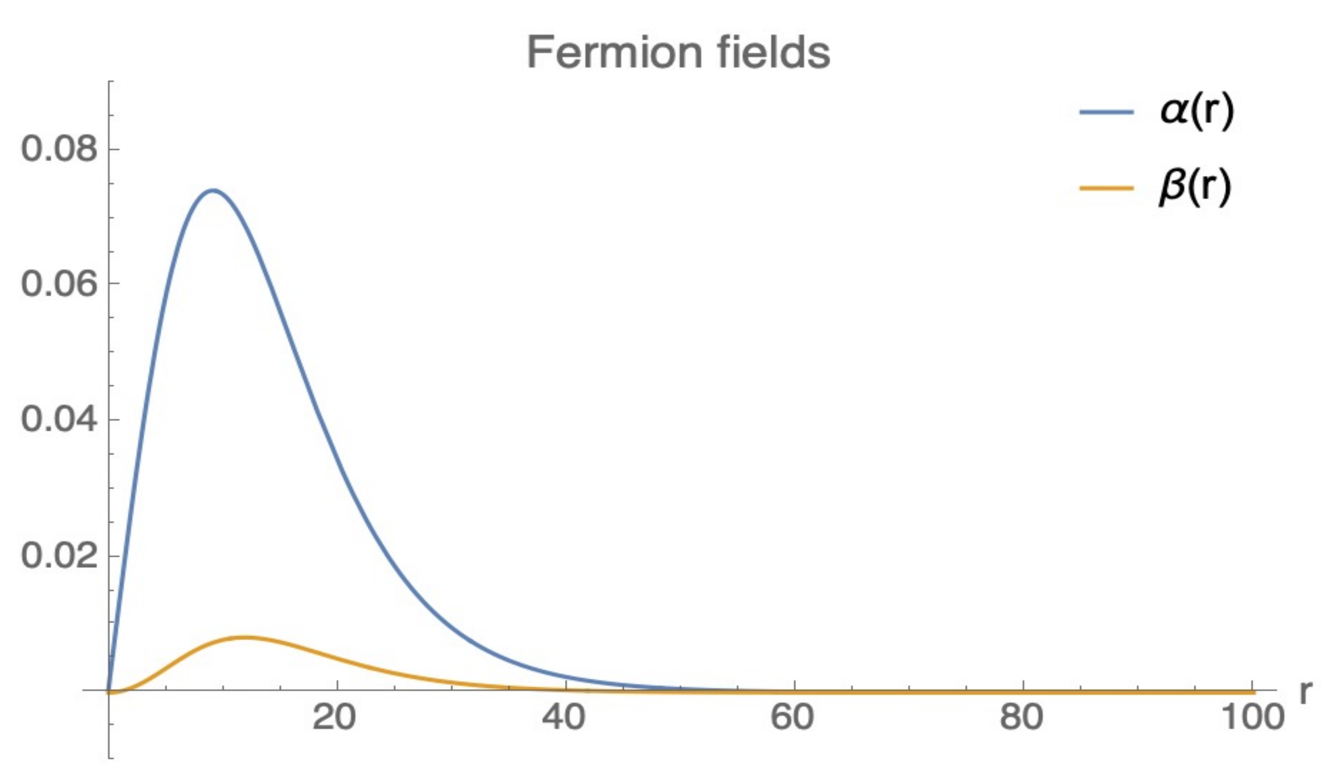

Fig. $1a$: Fermion fields for $\Lambda = 0.0001$ and $N = 2$ fermions.

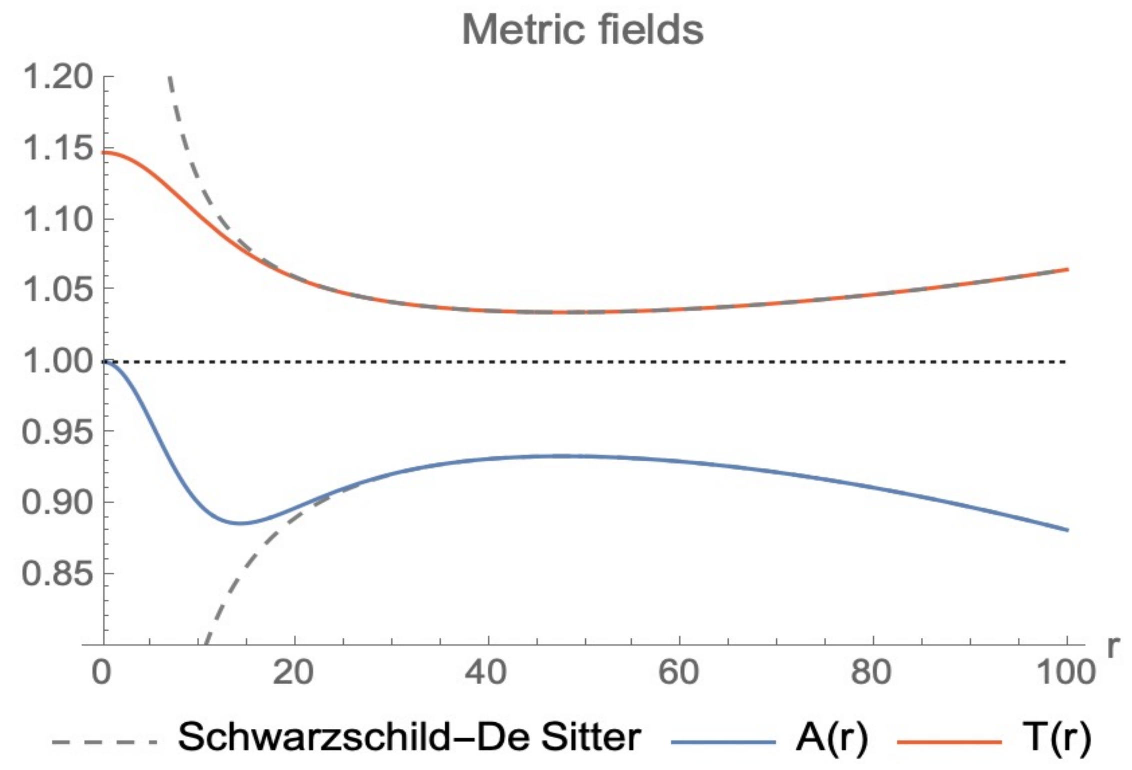

Fig. $1b$: Metric fields for $\Lambda = 0.0001$ and $N = 2$ fermions.

2) Reproducing a uniform matter background

We consider $3D$ space filled with a close packing of spheres, as displayed in Fig. 2. EDS lie at the centre of each ball.

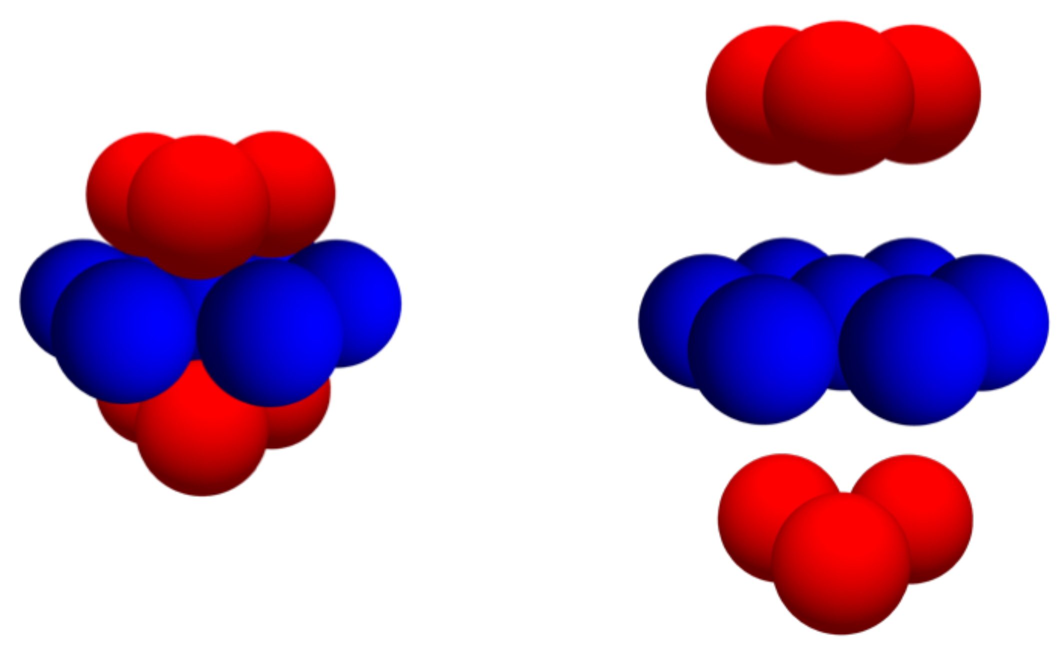

Fig. 2: Highly symmetric HCP arrangement. Each ball is surrounded by 12 balls.

We reproduce a uniform matter background which uniformity improves with the number of spheres that are computed, see Fig. 3.

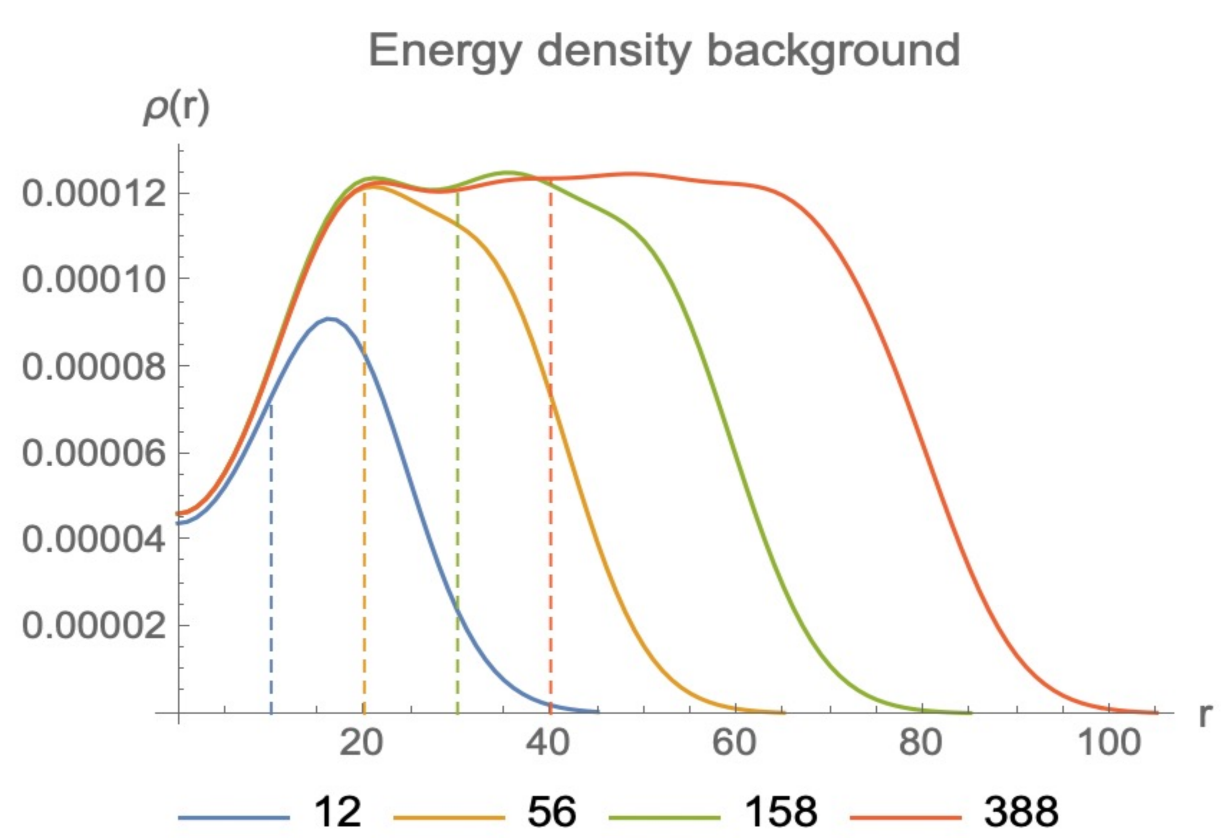

Fig. 3: This graph show the energy density background generated by different numbers of balls.

A uniform matter background is essential since uniformity and isotropy are fundamental assumptions of the Friedmann equations, with which the model could be expanded to describe an expansion process. Our results provide a good basis for further study on the phase transition behind fermionic solitonic condensation.

References

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