Convective Cloud Modelling

Aaron Carruthers
Supervised by David G. Dritschel
School of Mathematics & Statistics
University of St Andrews

LAIDLAW SCHOLARS

University of St Andrews

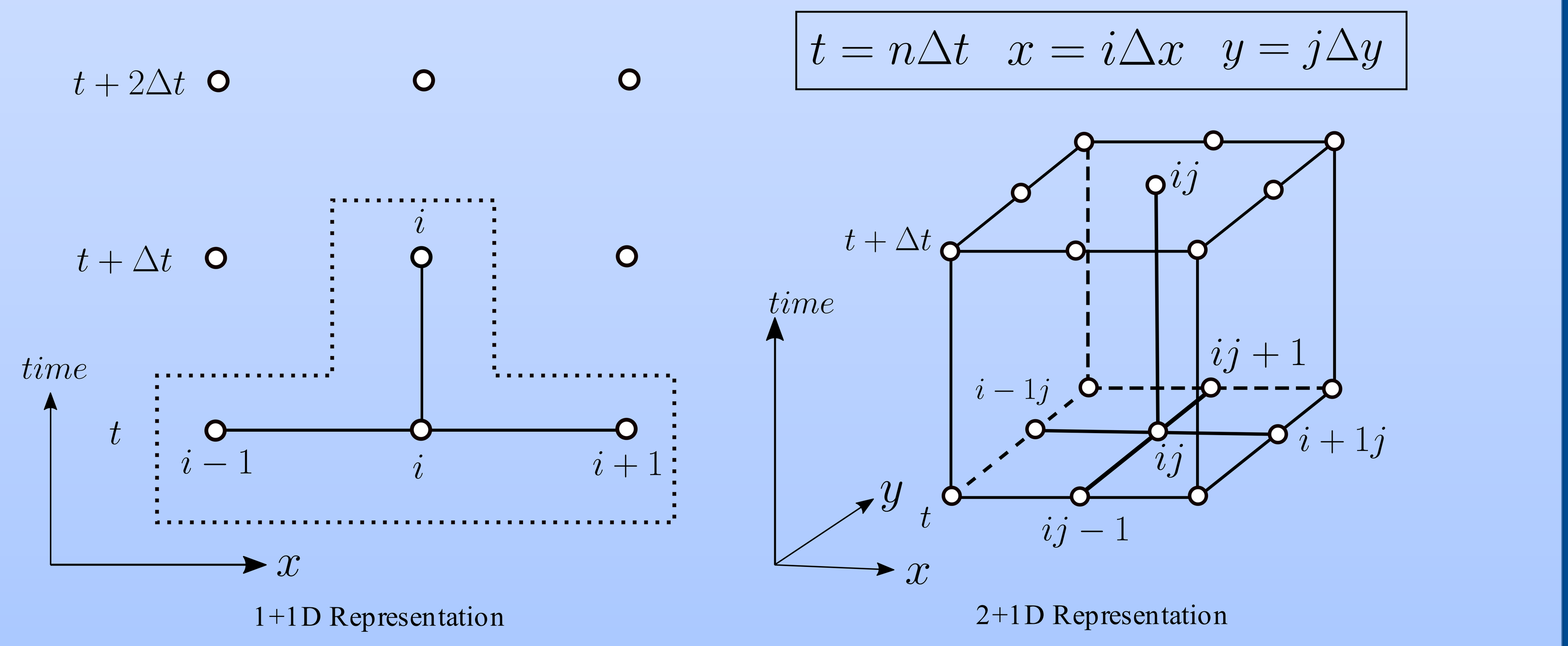
From plume to cloud

- Clouds form a natural part of our atmosphere and arise via thermodynamic and fluid mechanical processes that occur, particularly, in the upper troposphere. When water evaporates, the water vapor mixes with dry air to form moist packets which rise due to their buoyancy. This buoyancy is due to several factors that depend on the water content of the plume. [2]
- In this project we restrict ourselves to the effects of latent heating and cooling caused by the evaporation of liquid water and condensation of water vapor.
- A small plume of moist air is created near ground level and is attributed buoyancy due to the latent heating that went into creating it. This plume will rise forming a large vortex ring before crossing the condensation line. After crossing the line, the water vapor will condense. This releases latent heat, driving further convection and forcing the plume to rise higher. This creates the vertical structures common to cumulus clouds. As the plume rises further, its momentum is arrested due to its internal buoyancy eventually matching the increasing buoyancy of the background atmosphere. This increase in background buoyancy is due to a drop in density typical in the upper atmosphere. [2][3]

Discretizing Domain & Equation

- The equations need to be discretized such that they can be entered into code. A standard finite-difference scheme (FDS) was implemented and hence the domain space was broken up into a series of N by N nodes (32x32 typically). These nodes encode the data of each field with no data stored in between. This is a source of error and so it is wise to choose a differencing scheme that has the highest order truncation (error) term denoted $\mathcal{O}(\Delta x, \Delta y)$.[1][4][5]
- A Forward in Time Centered in Space (FTCS) scheme was employed for discretizing the equations of motion. This allowed for second order accuracy in the spatial domain, allowing for larger grid spacing and hence fewer computational nodes. [1][4]

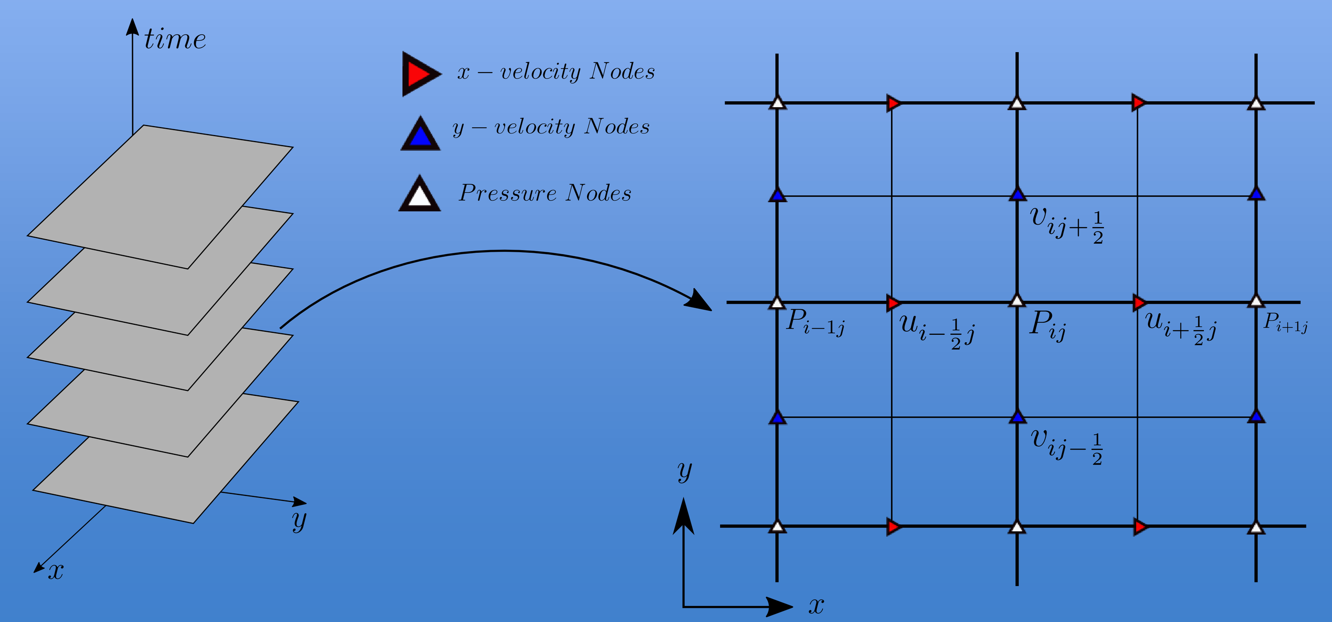
$$t = n\Delta t \quad x = i\Delta x \quad y = j\Delta y$$



$$\frac{\partial f(x,t)}{\partial x} \rightarrow \frac{f^n_{i+1} - f^n_{i-1}}{2\Delta x} \qquad \frac{\partial f(x,t)}{\partial t} \rightarrow \frac{f^{n+1}_i - f^n_i}{\Delta t} \qquad \frac{\partial^2 f(x,t)}{\partial x^2} \rightarrow \frac{f^n_{i+1} - 2f^n_i + f^n_{i-1}}{\Delta x^2}$$

Staggering the Grid

- To avoid decoupling of the pressure terms, which can occur when variations are oscillatory, a staggered grid scheme is used. This generates a grid for both components of velocity which are centered on the cell faces that exist between pressure nodes. All other independent fields, e.g. density, humidity etc. are also stored on the pressure nodes for simplicity. [4][5]



Analytical Model

$$\rho_0\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \mu_0\nabla^2\mathbf{u} + b\mathbf{e}_y$$

(1) Momentum Equation

$$b = b_l + \frac{gL}{c_p\theta_{l0}}q_c \text{ where } q_c = \max(0, q - q_0e^{-\lambda y})$$

(2) Buoyant Force where $\frac{gL}{c_p\theta_{l0}}$ contains the latent heat of condensation L

$$\left(\frac{\partial b_l}{\partial t} + \mathbf{u}\cdot\nabla b_l\right) = \mu_0\nabla^2 b_l$$

(3) Liquid-water Buoyant Force Transport Equation

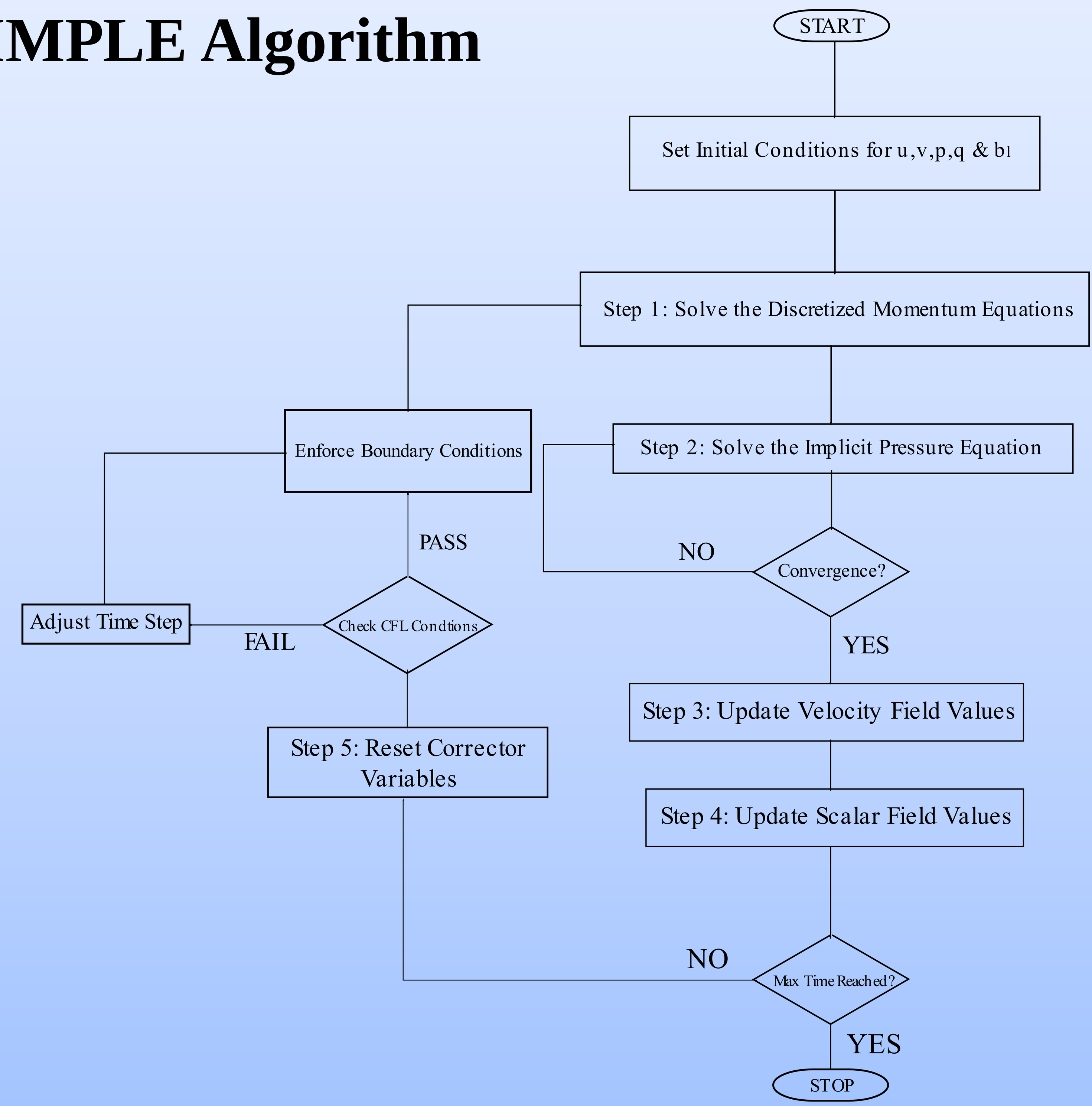
$$\left(\frac{\partial q}{\partial t} + \mathbf{u}\cdot\nabla q\right) = \mu_0\nabla^2 q$$

(4) Specific Humidity Transport Equation

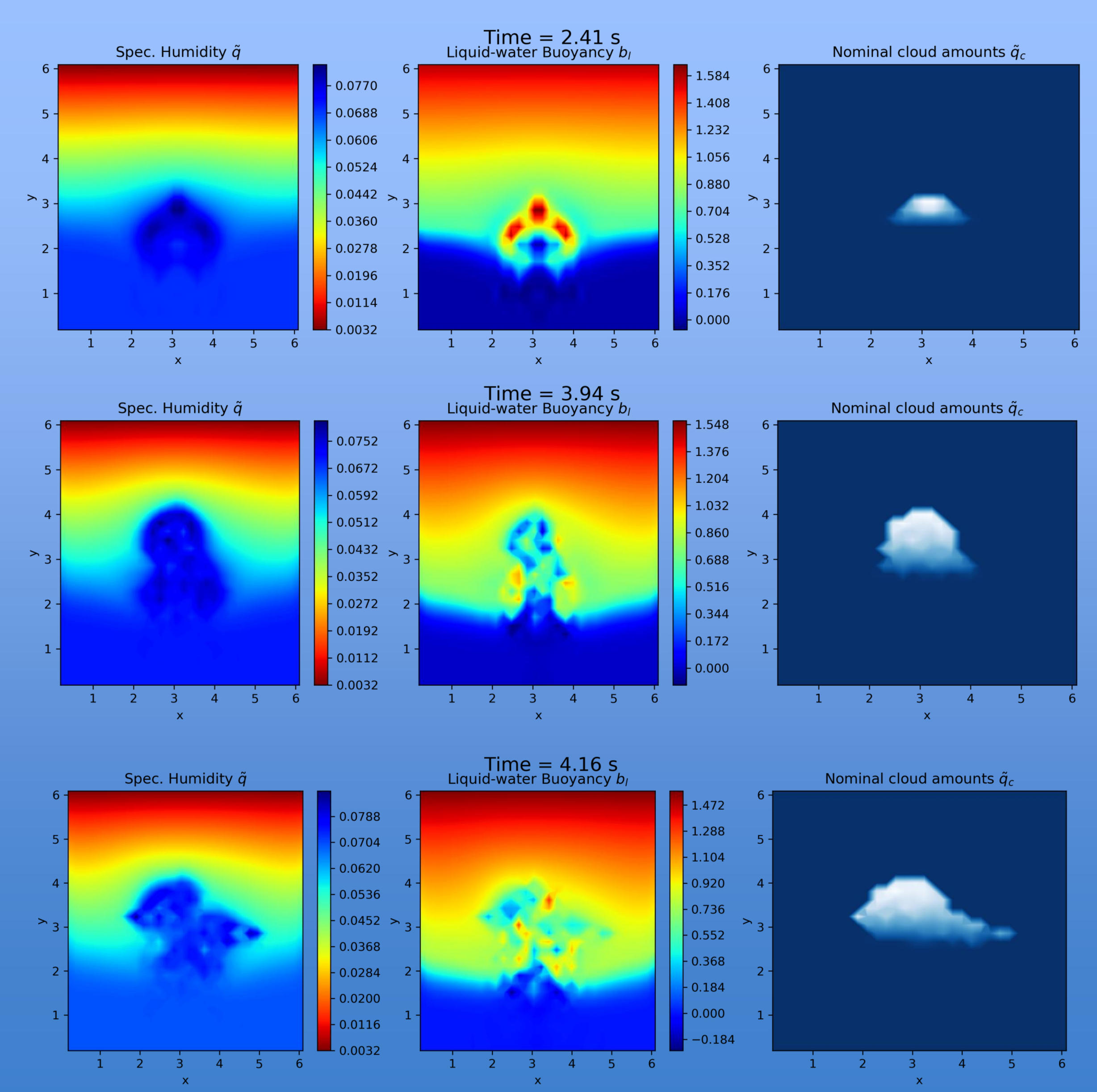
$$\nabla\cdot\mathbf{u} = 0$$

(5) Incompressibility Condition

A SIMPLE Algorithm



Results



References

1) A Front-tracking/Finite-Volume Navier-Stokes Solver for Direct Numerical Simulations of Multiphase Flows – Gretar Tryggvason – 2012 (Date Accessed: 06/2020 – 08/2020)
2) Dritschel DG, Böing SJ, Parker DJ, Blyth AM. The moist parcel-in-cell method for modelling moist convection. Q J R Meteorol Soc. 2018; 144:1695–1718. https://doi.org/10.1002/qj.3319
3) Betts, A.K. (1973), Non-precipitating cumulus convection and its parameterization. Q.J.R. Meteorol. Soc., 99: 178-196. doi:10.1002/qj.49709941915
4) J. D. Anderson Jr. – Computational Fluid Dynamics: The Basics with Applications – 1995
5) S. V. Patankar – Numerical Heat Transfer and Fluid Flow – 1980

Acknowledgements

I would like to take this opportunity to thank my supervisor Professor David Dritschel, as without his continued support and insights this project would not have been as successful as it was. I would also like to thank the Laidlaw foundation and Lord Laidlaw for providing me with this amazing opportunity and funding the project itself.