INVESTIGATING MATHEMATICAL PATTERNS IN MUSIC

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INTRODUCTION

Twelve tone music was established in the 20th century to challenge the traditional tonal functions of Western Classical music that had made it so pleasing to the ear.[1] It did this by eliminating bias towards any one note, thus resulting in the avoidance of a tonal key.

Mathematicians are fascinated with patterns, so one might ask whether we can still find patterns in twelve tone music despite the composer's attempt to avoid them. In other words, is human bias inevitably embedded in the process of composing music?

TWELVE TONE MUSIC

Twelve tone technique is a method of composing which involves:

1. Creating a tone row, an arrangement of the twelve tones such that each tone appears exactly once
2. Applying transformations to the tone row to create variations of it
3. Joining these variations together in some sequence to form the composition

The transformations traditionally used are prime, retrograde, inversion, and retrograde-inversion.

OBJECTIVES

The primary aim of this project is to find out whether composers using twelve tone technique use the full range of musical possibilities available to them, or whether they bias towards certain ones.

Other aims include:

+ Providing new pathways through which people can appreciate twelve tone music and abstract algebra
+ Demonstrating how pure mathematics can be applied to make discoveries in other disciplines
+ Encouraging realisation that there is more to music than 'how it sounds'

PROCESS

MATHEMATICAL INSIGHT

It turns out that the twelve tone transformations form an algebraic object called a **group**. Using group theory, we instantly see that there are more musical transformations we can also consider which are just as mathematically significant as the ones traditional to twelve tone technique.

TOTAL POSSIBILITIES

To make useful deductions from the data, we must find the total number of distinct tone rows. We derive general formulae for this by using knowledge from various areas of pure mathematics such as combinatorics.

ANALYSING MUSIC

We can then analyse the scores of various composers to see which tone rows and transformations they use, to then judge whether they bias towards certain possibilities or not.

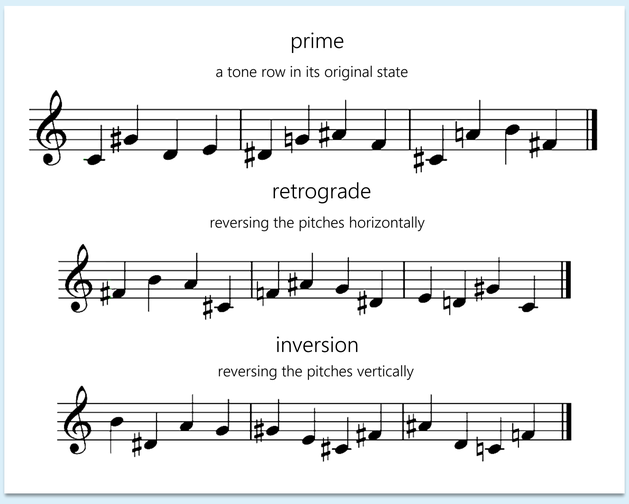

Figure 1: A diagram showing a tone row under the transformations prime, retrograde, and inversion.

FINDINGS

USE OF TRANSFORMATIONS

In addition to prime being the most used (which is expected), it was found **from the works studied** that:

+ Arnold Schoenberg least favoured retrograde-inversion
+ Anton Webern favoured retrograde-inversion, and used prime relatively less in proportion to the other transformations, compared to Schoenberg

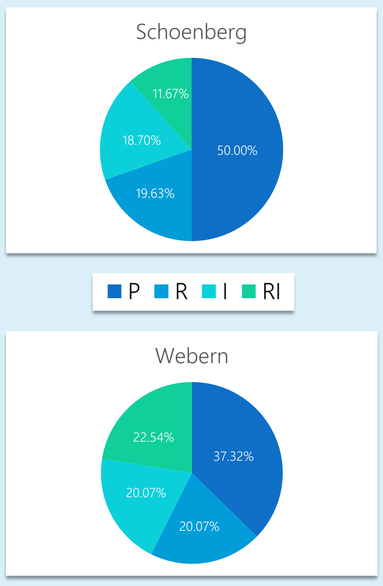

Figure 2: Pie charts showing the distribution of the transformations chosen by various composers in the works studied.

USE OF INTERVALS

In the works studied, semitones were heavily favoured in the prime tone rows chosen. Additionally:

+ Arnold Schoenberg favoured whole tonal (i.e. by 2 semitones) ascents
+ Anton Webern favoured semi tonal descents
+ Alban Berg favoured semi tonal and whole tonal ascents but to less of an extent compared to Schoenberg

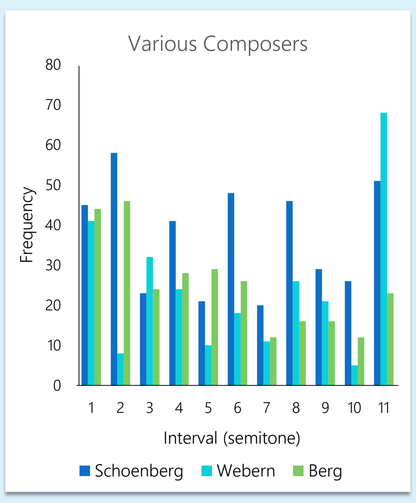

Figure 3: A bar chart comparing the occurrence of intervals in the prime tone rows chosen by various composers.

POSSIBLE ISSUES

Due to time constraints and limited access to sheet music, only a few composers and a limited range of their works could be studied, so the findings obtained are not representative of the overall compositional output of each composer. They are also not representative of the entire twelve tone music genre since each composer's method is unique to the composer.

SUMMARY

Twelve tone technique is a method of composing music which aims to avoid bias. However, from the research conducted, it was found that composers still showed bias in their work anyway.

It is important to note that the results obtained are not representative of the entire twelve tone music output due to various practical limitations, so further study to confirm these results for each composer, and explore possible explanations for them is highly encouraged.

Scan for more resources regarding my research project and experience as a Laidlaw scholar.

WHAT NEXT?

Possible directions for future research include:

+ **Analysing more works** by more composers to build a bigger picture and to confirm results (also explore whether later composers, who deviate from traditional twelve tone technique, use the additional transformations motivated by group theory)
+ **Looking into reasons** why composers may have biased towards the transformations and intervals they chose
+ **Extending the research question** to consider total serialism - twelve tone technique concerns pitch, but serialism generalises this to all musical elements such as rhythm (duration of notes), timbre (quality of sound due to the instrument used), dynamics (loudness of notes) etc.

REFERENCES

[1] Headlam, D., Hasegawa, R., Lansky, P., & Perle, G. (2001). Twelve-note composition. *Grove Music Online.* Accessed 5 Jul. 2020, from https://www.oxfordmusiconline.com/grovemusic/view/10.1093/gmo/9781561592630.001.0001/omo-9781561592630-e-0000044582.

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