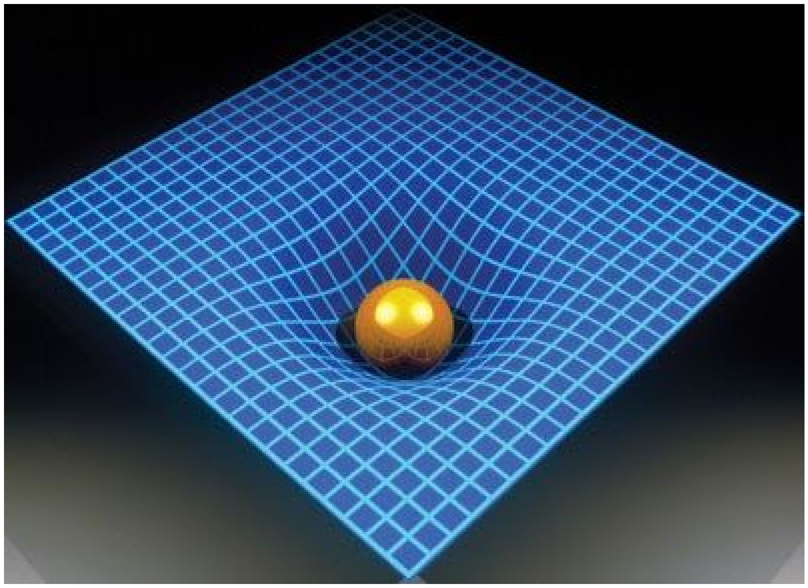
This study aimed to build a macroscopic analogue of a quantum scale system known as an FSY system. Multiple methods were explored and a model based on interconnected circuits with currents representing the important quantities was eventually decided on. This model has been partially made, however it requires more time to complete.

At the frontier of modern physics, the energy required to experimentally test much current theory is far beyond the technological capabilities of humans. To get around this issue physicists try to produce systems which obey the same mathematical equations, but are on similar scale to the world that humans live in. The best known, successful example of this is the modelling of the edge of blackholes or more precisely their hawking radiation which is the production of pairs of particles on the edge of a blackhole. One of these particles escapes, the other is sucked into the blackhole. This is the process through which blackholes decay. By definition, it is impossible for a measurement device to measure the trapped particle and for its information to escape the blackhole, thus a model is necessary for any study of such a system. To do this, various methods have been used. Such as through sound: a sound wave travels through a perfect fluid. Phonons (vibrations) are produced on this wavefront, one escaping the sound wave, one unable to escape. Other models focus on using light and superconductor circuits (called josephson inductor transmission lines) to produce very similar systems. The goal of this project is to produce an analogue for a system thought to model the inside of a blackhole called an FSY system.

An FSY system is more usefully defined as a pair of Dirac solitons, two electrically neutral particles bound together by mutual strong gravitational attraction, prevented from collapsing by the uncertainty principle. First explored by Finster, Smoller and Yau (hence FSY) in their 1999 paper, these systems are on the boundary of quantum mechanics (the very small) and general relativity (the very large, and physicists' best description of gravitation), with the gravitational field non-quantised. This is an interesting area, as physicists currently do not have a single theory (termed grand unification theory GUT) which governs both of these regions. Finding such a theory has the potential to be the greatest scientific breakthrough ever and thus any study into such simplified systems could be useful in the long run. However testing systems such as FSY is extremely difficult as they are extremely small and cannot therefore be probed with current technology (their size is one the same order of magnitude as the Planck length ~ 1.6*10^-35). In preparation for this work it was important to gain an understanding of Tensor Calculus, General Relativity, and Relativistic Quantum Mechanics with its extension into curved space time.

The first system considered was to have a graphene sheet as space time and the charges as particles. This model would be highly intuitive since it is possible to visualise the charges as masses and the graphene sheets' reactionary charge distribution as the curvature of the rubber sheet, similar to visualisations of General Relativity (**Figure 1**). This would be useful for teaching and developing intuition in such areas. However, during initial stages of this project it became clear that since graphene is very stiff, it is highly inflexible to manipulate if the values are of incorrect scale. It would also mean that it would be virtually impossible to achieve a singularity or anything of large gradient, so no real work was done into properly outlining how to build this model.

Figure 1



The second method considered, was the use of a non-linear optical device to simulate the particles as photons in the cavity. Since this is inherently quantum mechanical and an area where there is a large body of research available, it was initially thought to be promising. However since photon-photon interaction doesn't occur, various apparatuses would be required to allow there to be any gravitational analogue, so this model was discounted.

The third possible method of generating an analogue model for the system was actually inspired by a Hawking radiation model, where you have two Josephson inductor transmission lines parallel to each other, with the particles and framework being four different currents. This is a far less intuitive candidate because whilst it may be possible to make the system mathematically equivalent, it is physically very different. However, this type of setup has been shown to be capable of producing the kind of back-action of the particles affecting spacetime itself, which is required by the FSY system (but is not present in Hawking radiation analogues). This system is also inherently quantum mechanical, which is highly attractive, since it means that quantum effects, which need to be accounted for, will be present naturally. To achieve this however, you first must combine the four first order equations into one second order equation, in alpha and A. In order to achieve this, each possible combination of substitutions would need to be explored at a high level, to find the combination offering the most simple solution. The criteria used for deciding the ease of solution was to first check whether one would need to solve an equation before the first substitution; and second, whether after the substitution, one is left to solve a differential to be able to make the second substitution. By using the substitution T from FSY 1 into FSY 3 (and 2 into 4), and then using these to eliminate beta, it is found that the second order differential is a long and complicated equation. This would make designing a circuit to fit with it very difficult, since every term in the equation has to be individually encoded into the circuit. Thus, the circuit would be impractical and an inefficient way to create such a model.

The final model was chosen where each individual equation is modelled as a small circuit and connected to the circuits in such a way as to eventually form the complete equations. For the remainder of the paper the following will be applied

- a is the amplitude of the particle which will be called alpha,
- b is the amplitude of the antiparticle which will be called beta,
- A is the spatial curvature,
- T is the temporal curvature,
- r is the radius,
- m is the mass (eventually set to 1)
- w is a frequency (also eventually set to 1)

In this model however the radius from FSY is taken to be time in the model, i.e. the distance from the centre of the FSY system corresponds in this model to the time elapsed from the beginning of the run. The quantities a, b, A and T correspond to the identified current in their corresponding circuits.

For simplicity (of the circuits), the first circuit studies was actually the second FSY equation: $\sqrt{A}\, b' = (w\, T - m)\, a - b/r$ [Equation 1]. First the internal circuit which satisfies $b' \propto -b/r$ was found. This was relatively easy as exponential decay is a very common characteristic of circuits. It was decided to use an RC circuit (**Figure 2**). To implement the 1/r dependency, the capacitance C is allowed to change according to: C=t/(R c1), ie. C increases with time. Where c1 is a constant of proportionality from the simplification of the $\sqrt{A}$ term, R is the resistance and t the time. To then add the dependency on the current corresponding to alpha (the first term in Equation 1), a transistor is introduced into the beta circuit with base voltage originating from the alpha current. It will be placed next to a cell on a wire which is in parallel to a resistor (**Figure 3**). To extend this to be dependent on T, another similar transistor system will be added to the a current transistor system. Similarly to add the dependence on A, the entire system must be damped by 1/Sqrt[A], so a transistor in a region of current in which it has correct dependence will be placed on the main wire and the overall system damped with the magnitude of this damping coming from to the gate voltage which will come from the magnitude of the current in the A circuit.

Figure 2

(the component on top is the capacitor, the bottom is the resistor)

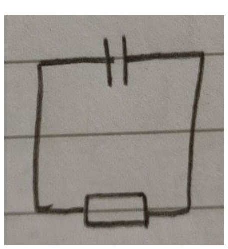
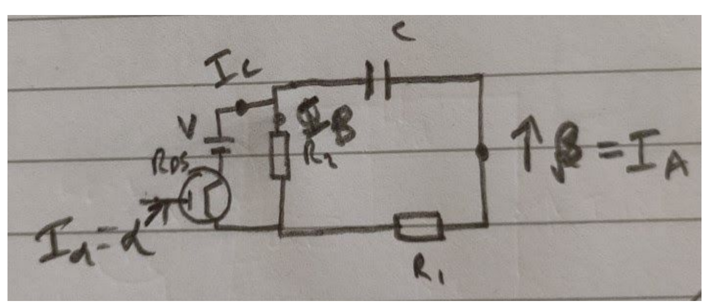
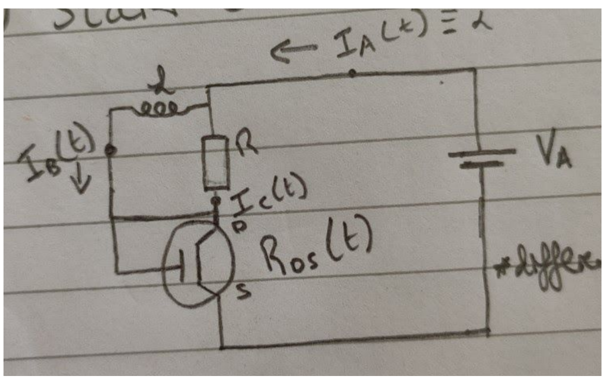


Figure 3



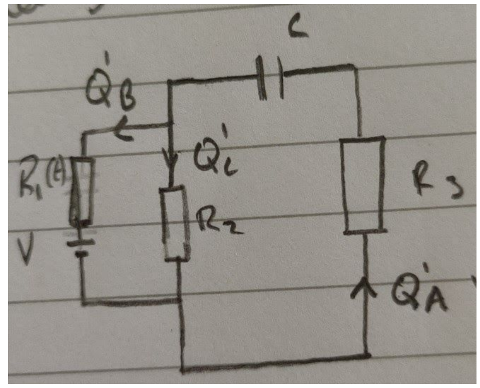
Next,was the turn of the first FSY equation: $\sqrt{A}\, a' = a / r - (w T + m)\, b$ [Equation 2]. Again, study began with looking for circuit which satisfies the internal equation: a' ∝ a, this requires for an internal gain in the circuit, ie the current increases by a rate proportional to the size of itself. In order to achieve this, a transistor with its base voltage coming from the circuit itself was added (**Figure 4**), this means that as the current in the circuit increases, (with constant resistance), the base voltage also increases, so the transistors resistance decreases, allowing more current to pass through the circuit and for the original current to increase. An inductor was also added as shown in figure 4, to cause a time delay to prevent a rapid feedback gain and the gain would last for a very short period, as the maximum current, ie. when the transistor has resistance of zero, would be reached. To then add in dependence on beta, something which causes anti proportionality would be added, one idea for this is a multi transistor system, where the current increasing in one transistor will decrease the current flowing into another transistor gate (possibly since they are in parallel). However this has clearly not been properly considered. Similar setups to those detailed in the paragraph about Equation 1, but with the added facet of the minus sign (likely being dealt with by introducing them to the double transistor system) will be added to this to integrate T and A.

Figure 4



To approach finding a circuit for equation 3: $A' = 1/r - A/r - 16\ w T^2 (a^2 + b^2)/r$ [Equation 3], one begins by starting with the A' ∝ - A/r dependency. This will solely require an RC circuit with the increasing capacitance similar to the one from Equation 1. To introduce the decreasing increase (ie a factor which increases the current, where the magnitude of this increase decreased in time, this comes from the 1/r proportionality term), a cell would be added with a variable resistor in parallel to the circuit (over a set resistor) (**Figure 5**), and vary the resistance according to a relation R1=(V t-R3)/(1+R3/R2). Where R1 is the variable resistor, R2 is the resistor which the cell is in parallel with, V is the voltage of the cell and t is the time elapsed. To add in the dependency on alpha, beta and T, first, a system like the double transistor one is needed, but where there were transistors with gate voltage from other circuits previously, a component such as the power of a bulb and photoreceiver pair with squared proportionality must be used.

Figure 5



The fourth FSY equation:
$2\,r\,A\,T'/T \;=\; A - 1 - 16\,\pi\,\omega\,T^{2}(a^{2} + b^{2}) + 32\,\pi\,T\,a\,b/r + 16\,\pi\,\omega\,T(a^{2} - b^{2})$ was not properly analysed. However once specifics of the previous equations have been ironed out, this will mainly just be a compilation of similar techniques.Though of course with some additional challenges, such as the cubic term in T, which will appear once you multiply through by the T from the left hand side. This will likely be solved by choosing the transistor (or other element) to be one which has cubic dependency in the range of currents in which the current T will sit. However a more elegant solution (of similar vein to the lightbulb type system for Equation 3) would be more satisfactory and accurate as this would only be an approximate solution.

Clearly there is much work left to be done to complete this model as has been detailed above, however a significant proportion has been achieved . Next steps would be to actually build the circuit, which should be achievable; to extend the circuit to the case of many particles (as in the research of the St Andrews group who works on the area); and to possibly even extend this to include an analogue for time evolution, however since time is already an analogue for radius in this model, it could be difficult to achieve.

I would like to thank my supervisor, Dr Chris Hooley, for the opportunity to work with him and explore this fascinating area of physics. I would also like to thank the entire Laidlaw team, especially Lord Laidlaw himself, who made the Laidlaw Scholarship program possible.

https://physics.stackexchange.com/questions/155547/visualizing-gr-spacetime-distortion-in-11d-spacetime-instead-of-2d-space figure 1