Research Proposal

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1 Title

Topological Origami: Material Properties from Geometry and Combinatorics

2 Project Description

This project will develop the understanding of origami like structures from condensed matter physics from the viewpoint of graph rigidity and combinatorics. Graph rigidity has roots dating back to a 1770 conjecture of Euler. Using modern terminology, it stated that a spherical polyhedron has its 3-dimensional 'extrinsic' shape determined by its 'intrinsic' face shapes. Over the next 200 years, this conjecture motivated the development of a body of results on the link between extrinsic shape and intrinsic metric information.

Much more recently, similar questions for repetitive origami patterns have been considered. Here, the typical behaviour is quite a bit different; a general repetitive fold pattern will move, and the eventual shape and nature of the allowed motions is stable to small changes. The phenomenon opens the possibility of designing interesting motions using crease patterns alone. In the first summer, I will explore the properties of triangulated patterns using inductive constructions from graph rigidity theory. I will then seek to use these "one step at a time" methods to design new examples.

During the second summer, the focus will shift from theory to construction. I will create basic models, both physical and computational, that can be easily reproduced. I will lead instructional sessions on how to construct models from common arts and crafts materials, giving young students a very friendly introduction to the otherwise complex study of geometry, topology and combinatorics.

3 Development and Impact

The aim of my project is to develop a deeper understanding of the topological mechanics of origami. Understanding the motions of origami, and related polyhedral surfaces, is a basic question in geometry that dates to Euler. Independent of the applications in construction, the questions I will research form an important part in the study of geometry and topology.

A successful outcome will be measured by the production of structures with desirable properties, such as materials, floppy at one edge, but stiff at another, that depend only upon fold patterns and geometry, not the material they are made of. This will be useful in construction, since the increasing scarcity of materials means we cannot indefinitely rely upon utilising their intrinsic chemical properties.

Initially, I will explore the topological properties of existing structures from condensed matter physics. I will then extend these findings to produce new theoretical examples with different properties. Finally, I will model these examples computationally, and build structures to demonstrate their uses and properties. There is scope to incorporate interdisciplinary aspects. Although I will tackle this project from a mathematical, rigidity theoretical perspective, I will work closely with concepts from physics and computing.

I recently participated in The Fields Institute's Winter School on Geometric Constraint Systems, with speakers from (but not limited to) École Polytechnique, M.I.T. and Ghent University; hence, I am developing an international network of academics in the field of geometry. Combined with my supervisor's collaboration network, there is potential to extend this project internationally.

I am fascinated by the concept of 'Maths Anxiety' – the feeling of tension, apprehension and fear when doing Maths. This project will allow me to apply my leadership skills in the second summer to give students an intuitive, hands on introduction to shape and motion, hopefully making maths seem less scary.

4 Resilience

The nature of this research project is largely theoretical, with the exception of running simulation software on my personal computer, and working with basic arts and crafts materials such as construction paper. Therefore, there is no requirement for any special equipment or lab access that could be made unavailable due to Covid-19 restrictions.

Additionally, one of the skills I have developed over the past year is my ability to work virtually and independently. Since the first lockdown in March, I have organised and chaired committee meetings for St Andrews University Maths Society (SUMS), held a seat on a Community Working Group with the objective of improving the experience of students during a largely online semester, and maintained my academic achievement given the shift to online learning. Therefore, I am confident in my ability to complete my research project within Covid-19 restrictions.