DESIGNING RESEARCH-BASED INTERACTIVE VISUALIZATION TO SUPPORT LEARNING IN QUANTUM MECHANICS

To Create a Stimulation as Part of the QuVis (Quantum Mechanics Visualisation Project) to Improve Student's Understanding of Quantum Uncertainty and Classical Uncertainty.

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Word Count – 3000

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1. A Brief Outline of the Research Project

This Project was funded by the Laidlaw foundation created by Lord Laidlaw. I would like to thank Lord Laidlaw for his generosity. I would also like to thank my supervisor, Dr Antje Kohnle for her guidance and understanding during my time on this research project.

The aim of the research project was to create an interactive stimulation to enhance student's understanding of the Quantum and Classical uncertainty using the quantum concept of spin. This Research Project is part of larger project called QuVis (the Quantum Mechanics Visualisation Project)[1] The Stimulation's target audience and content is in accordance with the third-year module at the University of St Andrew's Physics and Astronomy[2]department, called Quantum Mechanics 2(PH3062).

The layout of the Project was as follows. The research project commenced on the 31st of May 2021. The first four weeks of the project were dedicated to creating the stimulation using Mathematica, jQuery, and JavaScript. The last final two weeks of the project were spent conducting and preparing student interviews. The Interviews carried on throughout August. Between the 9th of July and 29th August 2021, the results from the interviews were analysed. Between each interview, there was some alternations made to the simulation.

2. The Physics Behind the Simulation

The aim of the simulation was to help enhance student's understanding of Quantum and Classical uncertainty. This next section will attempt to explain quantum and classical uncertainty for someone with no previous training in mathematical or physical sciences.

From an outer's perspective, physics is the science that enables humans to understand how the world works and our understanding enables humans to predict and harness nature. However, nature laws are a lot more intricate than one size fits all (for the time being at least). This brings us to the terms 'Classical Physics' and 'Quantum Physics'. The pivotal difference between the

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[1] https://www.st-andrews.ac.uk/physics/quvis/about.html
[2] https://www.st-andrews.ac.uk/physics-astronomy/

two is their deterministic nature. To say that a law is deterministic, is to say that you can predict confidently the outcome by looking at all the factors preceding the outcome. For example, if a ball is thrown the factors would be things such as the velocity[3] of the ball. For variables such as the speed of the ball as these variables change with time, they are said to be dynamic variables. A static variable would be the mass of the ball as this doesn't change with time. How the dynamic variables change with time is described by equations of motions which are shown below.[4]

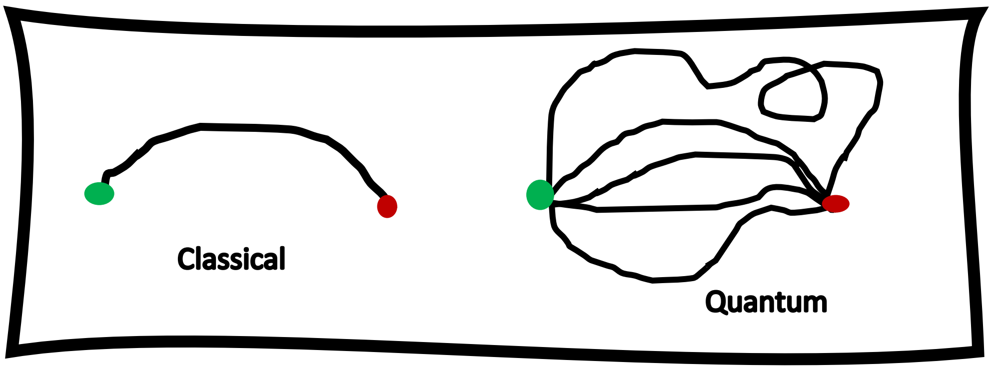
$$s = vt$$

[Eqn.1]

$$a = \frac{w - u}{t}$$

[Eqn.2]

In Classical Physics, the equations of motion tell us the precise position of the ball at any later time. To clarify, at any given time, the equations of motions can tell us the path of the particle. But, from the Quantum Mechanics viewpoint, the particle can take any path simultaneously from where the ball was thrown to where the ball lands. This is shown in **Image 2.1** where for a classical trajectory, the position of ball can be defined at any time between the green circle and the red circle. Whereas according to quantum physics, the ball is simultaneously following all the trajectories between the green circle and the red circle.



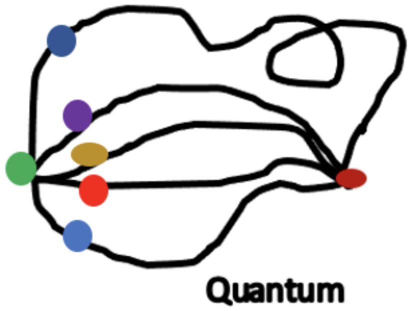
[Image 2.1]

The quantum state of a system is the probability (how likely) each possible measurement of the system is. For example, the quantum state of a 6- faced

[3] Velocity is a vector quantity in which it encompasses the speed of an object and the direction of the object speed for example a beach ball being thrown at 10 metres per second towards the geographical north would have a velocity of 10 metres per second North.

[4] s stands for displacement (m), *v* stands for velocity (ms^{-1}), *t* stands for time (s), *a* stands for acceleration (ms^{-2}), w stands for final velocity (ms^{-1}), and *u* stands for initial velocity (ms^{-1}).

dice would have a probability of 1/6 for each possible measurement which are 1,2,3,4,5 and 6.



[Image 2.2]

In **Image 2.2**, the colourful dots are all the possible measurements outcomes at time t. When a system has more than one possible state such as this example (there are 5 possible positions the ball can be found). It is said to be in a superposition of states, this is because the path can be simultaneously in all five positions at once.

Dot	Probability
	0.1
	0.2
	0.25
	0.35
	0.2

[Table 2.1]

The key thing to know about Quantum Mechanics is that the possible measurements of system are restricted to discrete values (this is called quantisation this is where the why it’s called quantum mechanics), these are called the eigenvalues of the system.

So, in the above example, there are 5 possible measurements (dots), hence 5 eigenvalues which each have a corresponding probability. The probability denoted in the table 2.1 is the likelihood that when the system is measured the outcome would be that dot for example if the state in Image 2.2 is measured, there would be a 35% chance of observing a red dot.

In quantum mechanics, when a measurement takes place, the superposition state ‘collapses. When, a superposition state collapses into one of its measurement outcomes. Any future measurements for the same property of the system would reveal the same measurement as the system would have collapsed into one of its eigenstates.

To understand in terms of the dot example. If we measured the property of the system, which in this case is the dot colour, we get a red dot any future successive attempts to measure the ‘dot’ property will yield a red dot. This is because the quantum state is no longer in a superposition of different coloured dots and their corresponding probabilities but it’s now only in a single dot state, the red dot eigenstate. This is where quantum uncertainty comes in. When a state is in one of its eigenstates, any successive measurements will yield the same results hence the quantum uncertainty for the successive measurements will be zero as we can 100% say what the result would be. In dot terminology, if we measure a red dot initially, we can state with zero quantum uncertainty that the next measurement will yield a red dot.

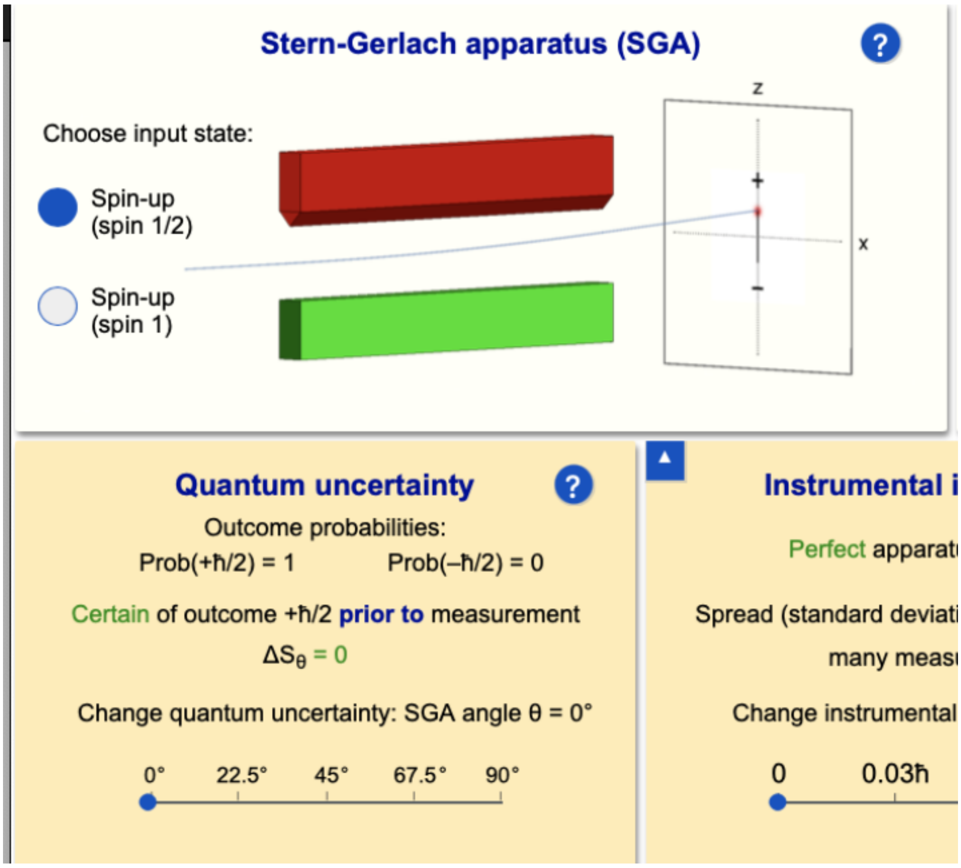
Quantum uncertainty would be non-zero when a state is in a superposition state as we can’t say with precisely what eigenvalue would be observed when measured.

The Expectation value is the weighted average of each possible measurement outcome. If a quantum state was in one of its eigenstates. The expectation value would be the same as the eigenstate as it’s having 100% probability of being in that state (1 x eigenvalue = eigenvalue).

In the simulation, the property being measured is spin[5]. For a spin 1/2 particle, the discrete measurable outcomes are $+\frac{\hbar}{2}$ and $-\frac{\hbar}{2}$. The Input state for the Stern-Gerlach experiment in the simulation is $+\frac{\hbar}{2}$ (it is in an eigenstate). However, as the Stern-Gerlach apparatus rotates its angle (see Image 2.3), the

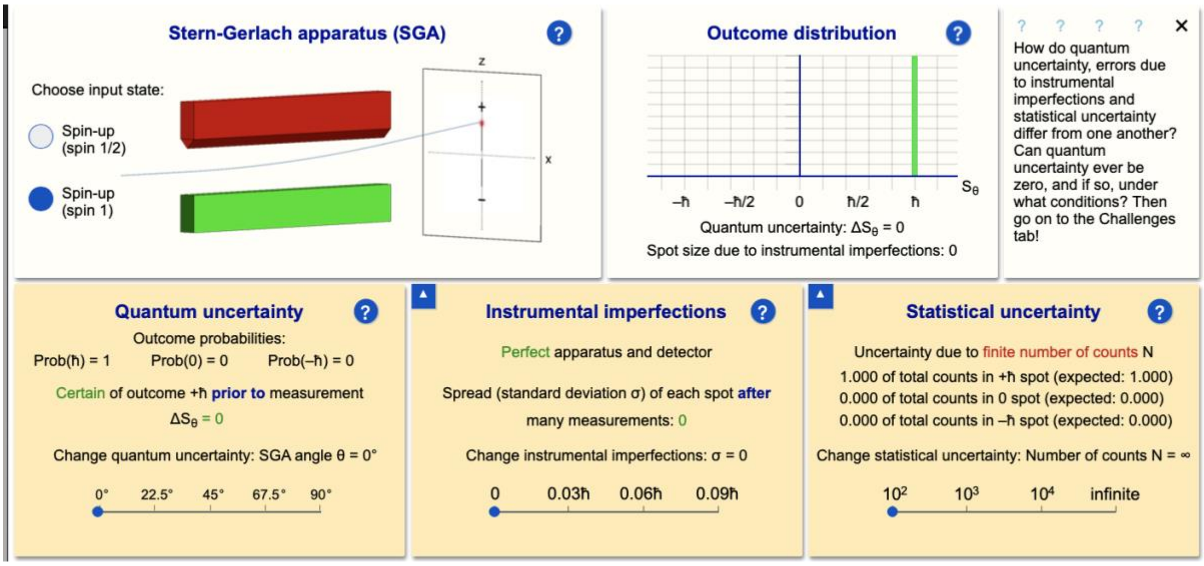
[5] For more information about spin; https://www.scientificamerican.com/article/what-exactly-is-the-spin/

quantum state is no longer in an eigenstate but a superposition state. A simple way of thinking about it is that as you change the angle of the apparatus, you change the probabilities of each measurable outcome.

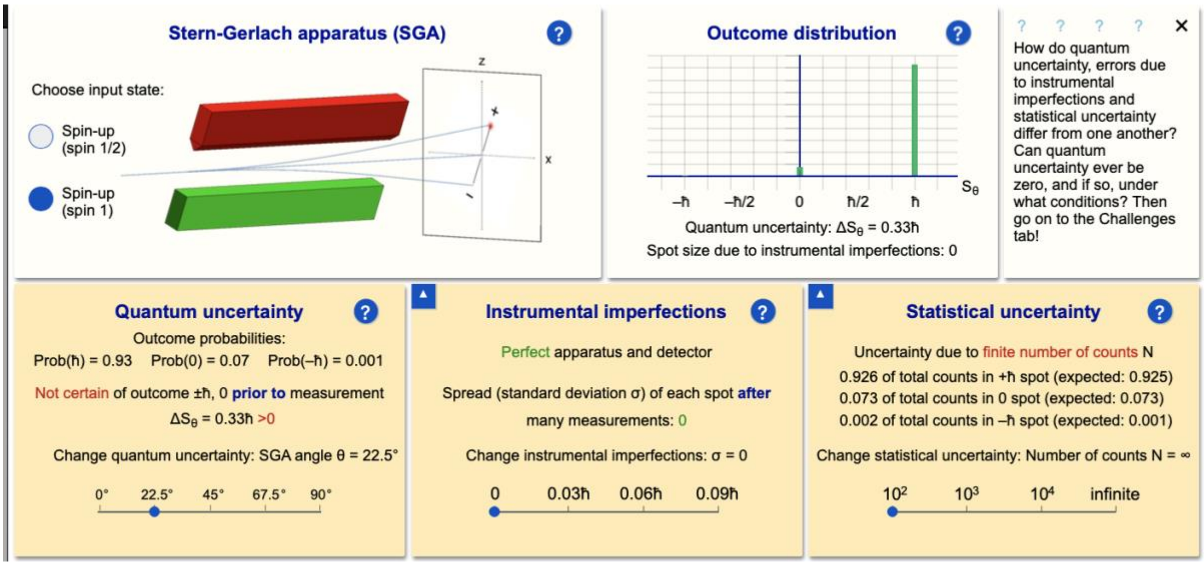


[Image 2.3]

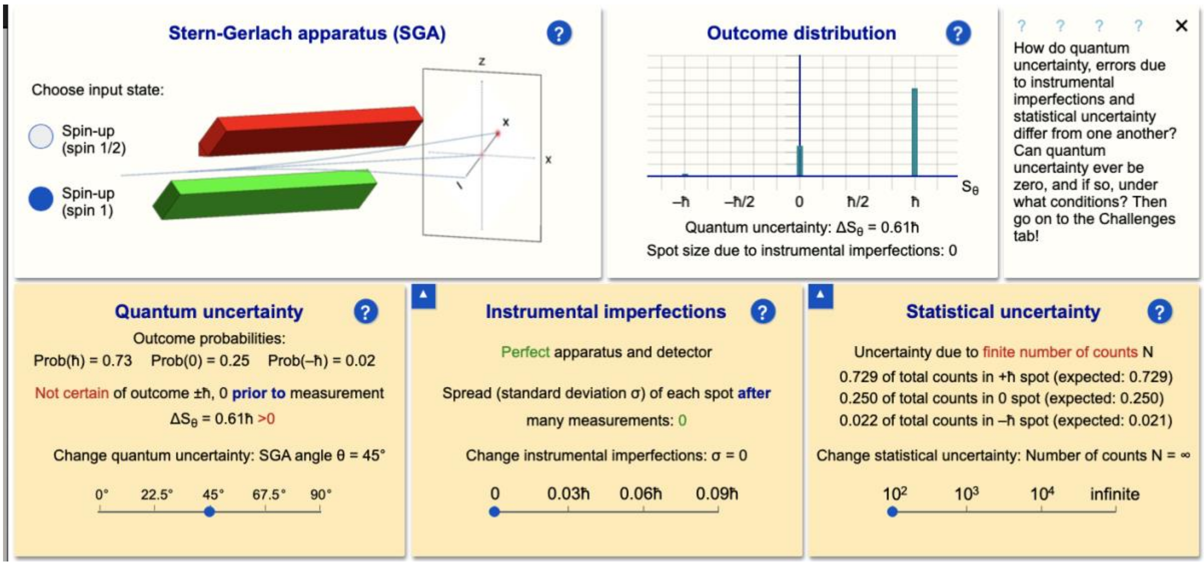
Image 2.4a to 2.4e demonstrate the impact of changing the theta value from 0 degrees to 90 degrees. An important thing to observe is the change in the intensity of the dot spots on the screen in the top left-hand panel and the histogram depicting the outcomes. The input state in the below images is in the spin up ($+\hbar$) eigenstate of Spin-1 particle in which the eigenvalues are $+\hbar$,0 and-$\hbar$.



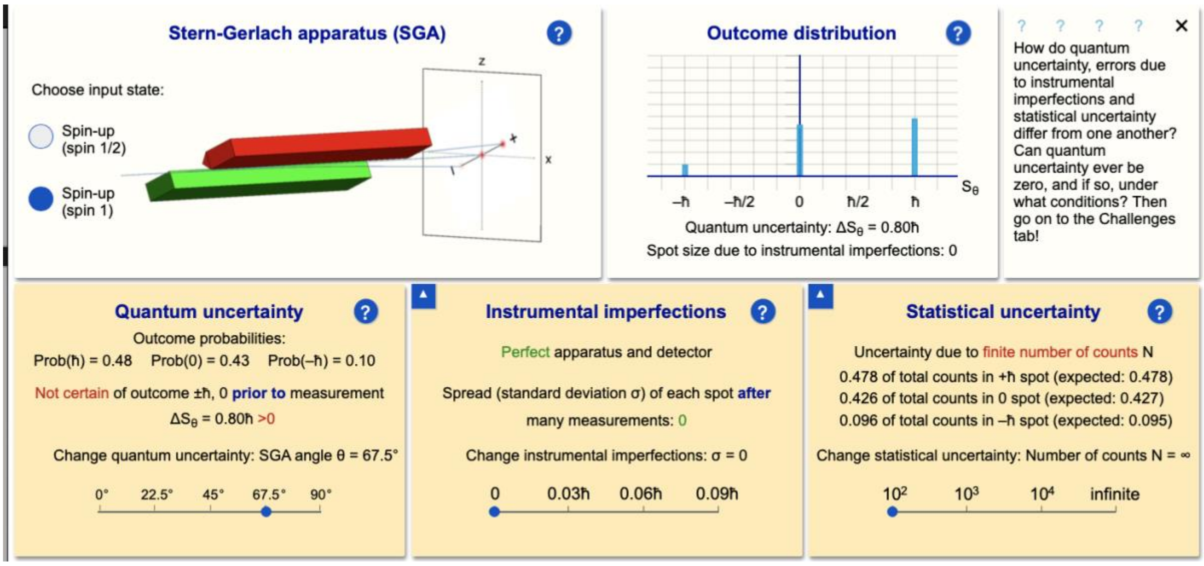
[Image 2.4a- 0 Degrees]



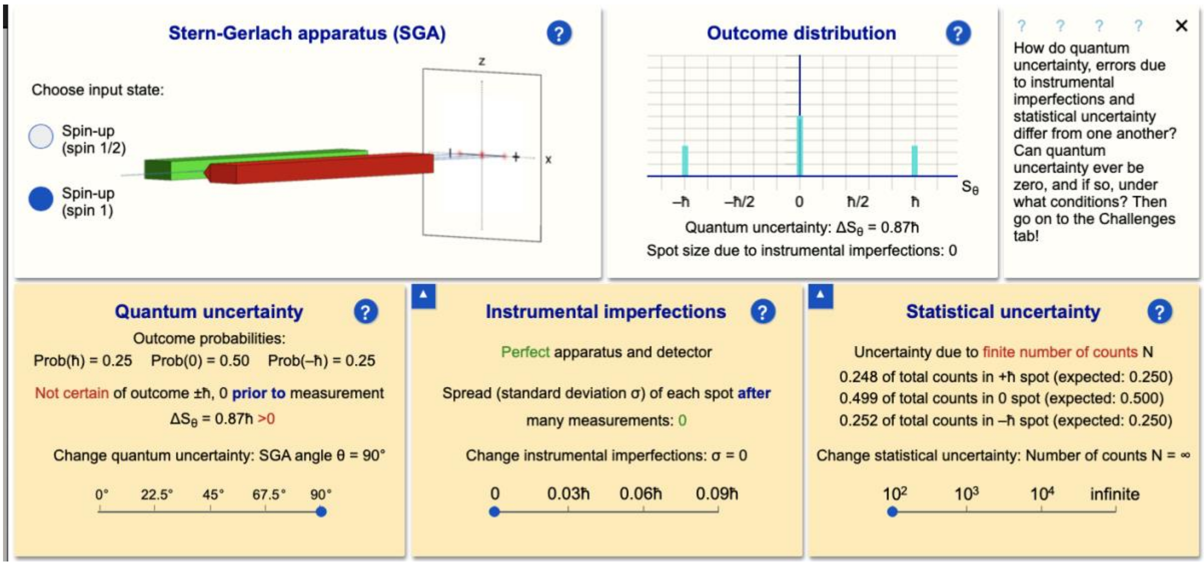
[Image 2.4b - 22.5 Degrees]



[Image 2.4c- 45 Degrees]



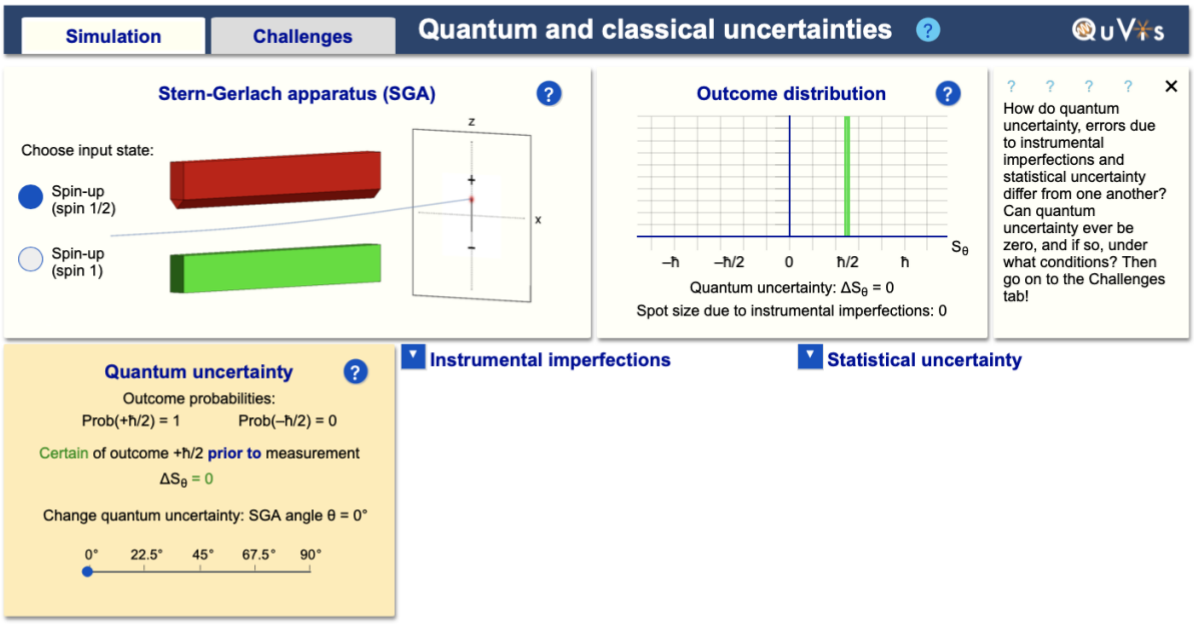
[Image 2.4d- 67.5 Degrees]



[Image 2.4e- 90 Degrees]

3. The Creating the Simulation

The Simulation was created using Mathematica, jQuery, and JavaScript. Image 3.1 shows a screenshot of the simulation.

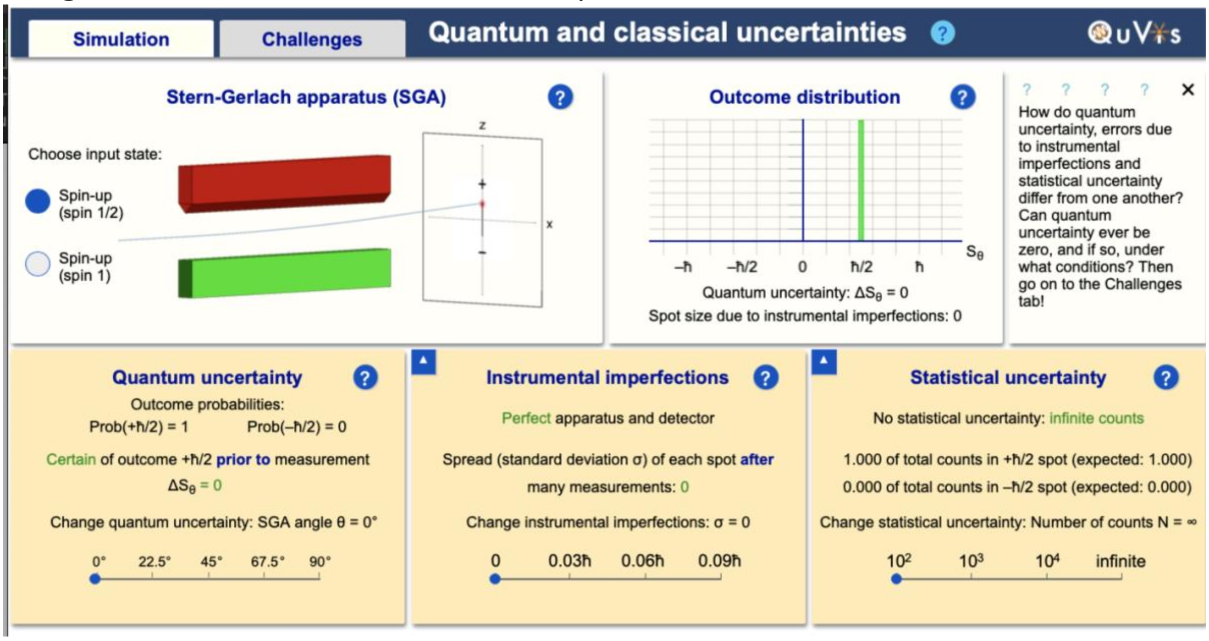


[Image 3.1][6]

[6] https://www.st-andrews.ac.uk/physics/quvis/test/uncertainty.html

The Stern-Gerlach apparatus was created using Mathematica with the help of Aly Gillies.

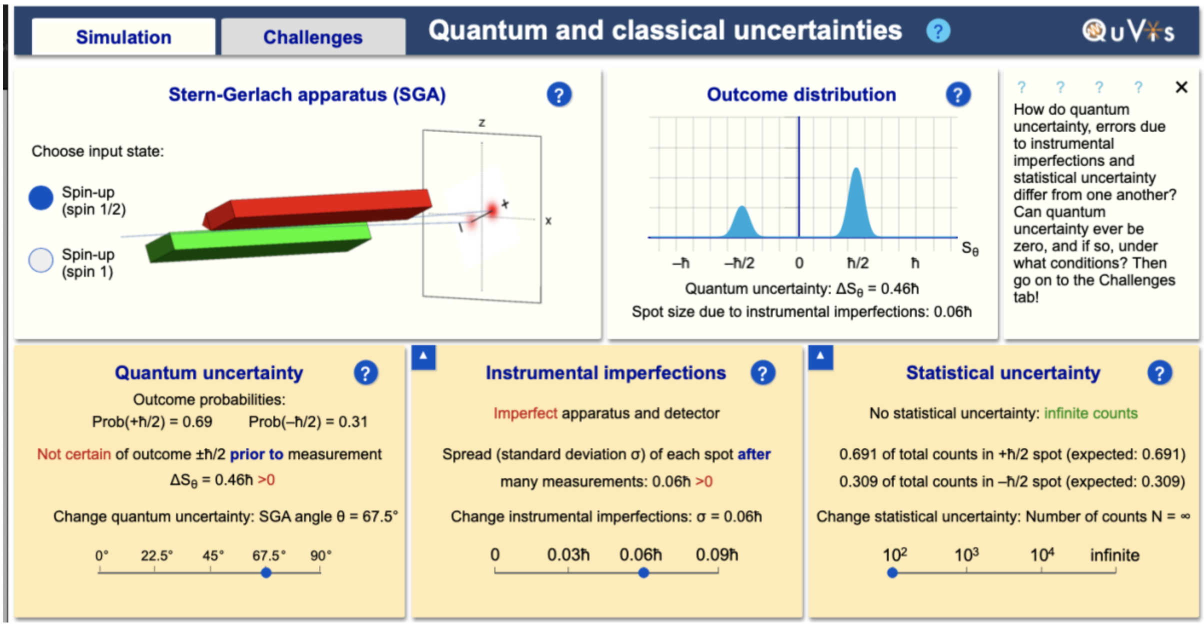
Certain features such as the drop-down arrows were included to allow the student to navigate at their own pleasure and not be overwhelmed by the influx of features. The drop-down bars are initially down. Image 3.1 shows an Image of the simulation when the drop-down bars are down.



[Image 3.2][7]

The bottom left panel enables the student to change the angle of the apparatus which changes the composition of the quantum state. Initially the quantum state is in the eigenstate of spin-up particles for both Input states (Spin 1 and Spin ½). The Instrumental Imperfections change the width of the narrow peak, so it appears like in Image 3.3 where in the outcome distribution panel the peaks have become broader and no longer resemble a narrow line.

[7] https://www.st-andrews.ac.uk/physics/quvis/test/uncertainty.html



[Image 3.3]

4. Methodology of the Interviews

There were five students who took part in the interviews. All of which completed Quantum Mechanics 2 (PH3062) at the University of St Andrew's School of Physics and Astronomy during the Martinmas Semester 2021. The students came from various degree paths which included Experimental Physics, Theoretical Physics, Joint Honours (Theoretical Physics and Applied/Pure Mathematics) and Astrophysics. Each student was rewarded £10 amazon voucher in return for their time.

The Interviews were conducted over Teams and were an hour long. The Interview were split into five parts. For each section, the student was asked to talk through their thought process and write their answers on separate piece of paper for each section. At the end of the interview, they were asked to send over a pdf of all their written answers. They were also asked not to later their answers top previous questions during the interview.

For the first section before they were shown the simulation, the student was shown a pdf with Pre-Simulation Questions (See Image 2).

Pre-Simulation Questions

1. **Consider a spin ½ particle, in a state where the probability of outcome $S_z = +\frac{\hbar}{2}$ is 0.75, the probability of outcome $S_z = -\frac{\hbar}{2}$ is 0.25, the expectation value $\langle \hat{S}_z \rangle = 0.25\hbar$ and the quantum uncertainty $\Delta S_z = 0.43\hbar$. Sketch a histogram of the outcomes depicting all of these values.**

2. **Do you agree or disagree with the following statement? Explain.**

   "For a given quantum state $|\psi\rangle$ the quantum uncertainty tells you the uncertainty after measurement, not before measurement?"

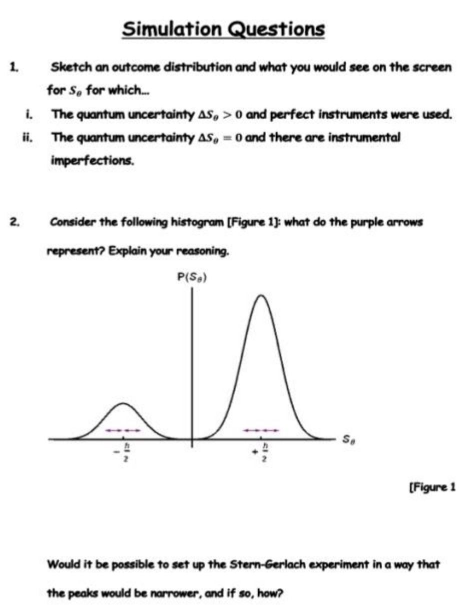
3. **Can quantum uncertainty ΔS_z for the spin component S_z ever be zero? If so, under what circumstances?**

[Image 4.1]

Next, the student was sent a link to the Quantum and Classical Uncertainty Simulation[8]. The student was asked to interact with the Simulation and talk through their thoughts on the simulation.

The student was then shown a PDF with the 'Simulation Questions' (Image 4.2 & 4.3). During this set of questions, the student was allowed to keep the simulation open to help them answer the questions. The Simulation Questions were used to help the student engage and fully understand the simulation and resembled that of a typical PH3062 tutorial question.

[8] https://www.st-andrews.ac.uk/physics/quvis/test/uncertainty.html



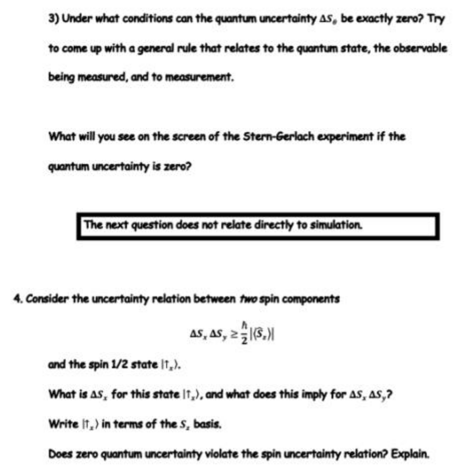
Simulation Questions

1. Sketch an outcome distribution and what you would see on the screen for S_θ for which…
   i. The quantum uncertainty $\Delta S_\theta > 0$ and perfect instruments were used.
   ii. The quantum uncertainty $\Delta S_\theta = 0$ and there are instrumental imperfections.

2. Consider the following histogram [Figure 1]: what do the purple arrows represent? Explain your reasoning.

[Figure 1

Would it be possible to set up the Stern-Gerlach experiment in a way that the peaks would be narrower, and if so, how?



3) Under what conditions can the quantum uncertainty ΔS_θ be exactly zero? Try to come up with a general rule that relates to the quantum state, the observable being measured, and to measurement.

What will you see on the screen of the Stern-Gerlach experiment if the quantum uncertainty is zero?

The next question does not relate directly to simulation.

4. Consider the uncertainty relation between *two* spin components

$$\Delta S_x \Delta S_y \geq \frac{\hbar}{2}|\langle \hat{S}_z \rangle|$$

and the spin 1/2 state $|\uparrow_x\rangle$.

What is ΔS_x for this state $|\uparrow_x\rangle$, and what does this imply for $\Delta S_x \Delta S_y$?

Write $|\uparrow_x\rangle$ in terms of the S_z basis.

Does zero quantum uncertainty violate the spin uncertainty relation? Explain.

[Image 4.2] **[Image 4.3]**

The students were then asked a series of feedback question regarding the Simulation.

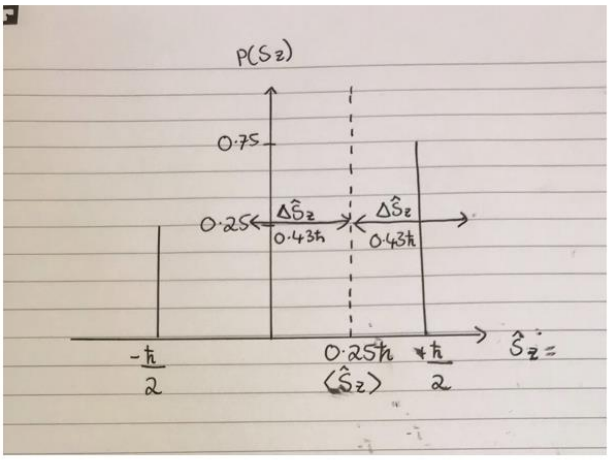
The students were then asked to close the simulation and go through the pre simulation questions again (Image 4.1). The student's answers to pre/post simulation question were then compared to discover how effective the simulation was at improving student's understanding of quantum and classical uncertainty.

5. Results of the Student Interviews

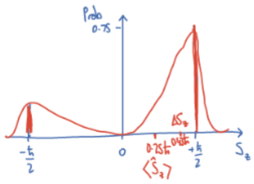
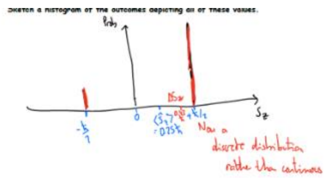
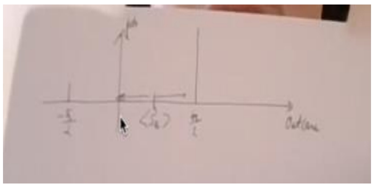
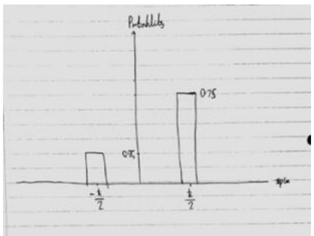
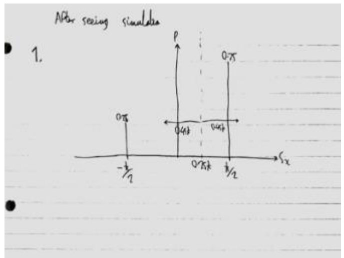
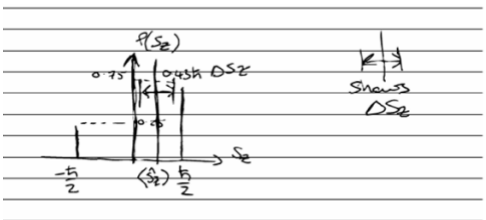
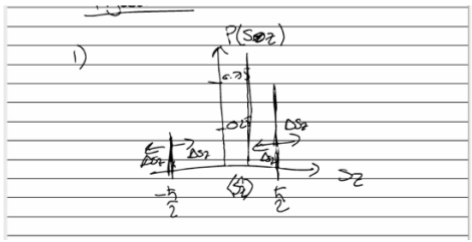
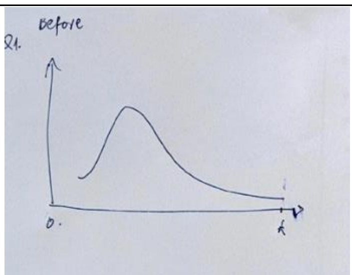
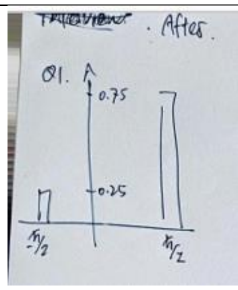
5.1 Pre/Post Simulation Question 1

For the Pre/Post Simulation Question 1 (Image 5.1) which asked students to depict graphically the outcomes, expectation values and the quantum uncertainty.
The students' pre and post simulation answers are shown in Table 5.1 (on next page).

1. **Consider a spin $\frac{1}{2}$ particle, in a state where the probability of outcome $S_z = +\frac{\hbar}{2}$ is 0.75, the probability of outcome $S_z = -\frac{\hbar}{2}$ is 0.25, the expectation value $\langle \hat{S}_z \rangle = 0.25$ and the quantum uncertainty $\Delta S_z = 0.43\hbar$. Sketch a histogram of the outcomes depicting all these values.**



[Image 5.1]

Student[9]	Pre-Simulation Answers	Post Simulation Answers
1	 [Image 5.2a]	 [Image 5.2b]
2	 [Image 5.3a]	No Change to Answer
3	 [Image 5.4a]	 [Image 5.4b]
4	 [Image 5.5a]	 [Image 5.5b]
5	 [Image 5.6a]	 [Image 5.6b]

[Table 5.1]

[9] The students have been randomised to preserve anonymity.

It is clear to see that the student's answers before the simulation differed from each other. The concept that appears to be at the centre of student's misunderstanding was how the expectation value was to be depicted on the histogram. Even the students, who correctly placed the expectation value on the x-axis in their pre-simulation answer were hesitate. Image 5.6a, the student started to draw Gaussian wave but then stalled when they considered the expectation value. Image 5.4a, the student also stalled. For the students who didn't feel confident with the expectation value placement, they also struggled with connecting the quantum uncertainty to the values in the histogram.

The students whose histograms were mostly correct (Image 5.5a, Image 5.3a) used a mathematical formula (Eqn.2) to help them understand the connection between the expectation value and quantum uncertainty.

$$\Delta\hat{S}_x = \sqrt{\langle{\hat{S}_x}^2\rangle - \langle\hat{S}_x\rangle^2}$$

[Eqn.3]

These same students were wary to extent the quantum uncertainty error bars beyond the outcomes. This suggests that although using mathematical formulas to help them get to the correct answer, their conceptual understanding of quantum uncertainty was less confident and conflicted with what the mathematical formulas told them.

The student's post simulations answers appear a lot more consistent. The most significant improvement would all the students drew straight line rather than gaussian waves. 4 out of the 5 students drew the expectation value in the correct place. However, there was still inconsistency in the inclusion of the quantum uncertainty with one student neglecting expectation value and quantum uncertainty altogether (Image 5.6b), continuing to mark quantum uncertainty as a point on the x axis (Image 5.2b). One student changed their initial correct depiction of the values to put the quantum uncertainty around the outcomes rather than the expectation value (Image 5.5b). This could be due to the simulation question 2, in which they were misled by purple arrows not extending across the entirety of the Gaussian, so they assumed that by cancellation. The inconsistently in the post- simulation answers could also be because, the only occurrence that the expectation value and quantum uncertainty is in challenge 5.

5.2 Pre/Post Simulation Question 2

Image 5.16 shows the Pre/Post Simulation Question 2 with the correct reasoning. Table 5.2 shows the students pre and post simulation answers side by side.

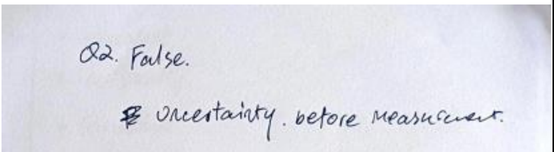
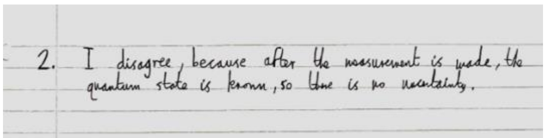
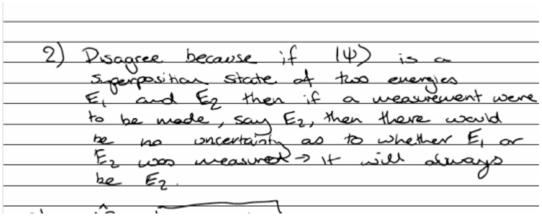
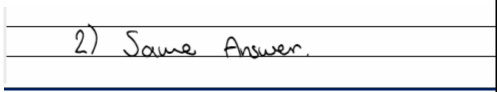
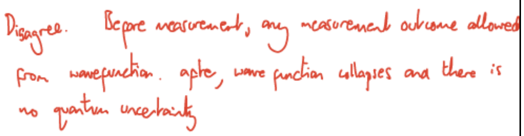
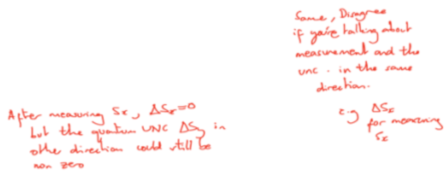
2. Do you agree or disagree with the following statement? Explain.

"For a given quantum state $|\psi\rangle$ the quantum uncertainty tells you the uncertainty after measurement, not before measurement."

Correct Reasoning

- **Quantum Uncertainty is related to the underlying probability distribution prior to measurement.**
- **Quantum uncertainty is not deterministic. Depending on the Quantum State, there is a non-zero probability that several different eigenvalues could result from the measurement. This results in an uncertainty inherently linked to theory.**
- **The Expectation value of the state is the weighted average of all possible eigenvalues, and the Quantum uncertainty is related to this.**

[Image 5.16]

Student [10]	Pre-Simulation Answers	Post Simulation Answers
1	 Q2. False. uncertainty before measurement. [Image 5.7.a]	 Q2. SAME. [Image 5.7.b]
2	 2. I disagree, because after the measurement is made, the quantum state is known, so there is no uncertainty. [Image 5.8a]	No Change to Answer
3	 2) Disagree because if $\|\psi\rangle$ is a superposition state of two energies E_1 and E_2 then if a measurement were to be made, say E_2, then there would be no uncertainty as to whether E_1 or E_2 was measured → it will always be E_2. [Image 5.9a]	 2) Same Answer. [Image 5.9b]
4	 Disagree. Before measurement, any measurement outcome allowed from wavefunction. after, wave function collapses and there is no quantum uncertainty [Image 5.10a]	 Same, Disagree if you're talking about measurement and the unc. in the same direction. e.g. ΔS_x for measuring S_x After measuring S_x, $\Delta S_x = 0$ but the quantum unc ΔS_y in other direction could still be non zero [Image 5.10b]
5	**'Disagree, once you measure a quantum state, you will collapse into an eigenstate of whatever you were planning to measure'[11]**	No Change to Answer

[Table 5.2]

[10] The students aren't in the same order as shown in Table 5.1
[11] Due to change in interview technique, answer would spoke out loud.

Table 5.2 shows that majority of the students kept to their original answer with one exception being student 4 in which they changed their answer (see Image 5.20b) Although they stuck with disagreeing with the statement. They appeared to be using what they learnt in Simulation Question 4 in which made students to consider the different components of spin (spin along different direction, for example a vertical spinning top rotating on the spot would only have a z-component of spin however if the spinning top was spinning at an angle, it would also have a x and y components of spin).

This Student's reasoning is partially correct though due to the incompatibility of spin x, y and z components having zero uncertainty for one spin component doesn't make the uncertainty zero for the other spin components. However, quantum uncertainty tells you about the uncertainty after measurement not before. Regarding the different spin components, you would only be able to measure one component with precision at a time so technically measuring the x-component of spin, you would not be measuring the y-component of spin.

Overall, all the students commented that the simulation made them a lot more confident in their earlier answers.

5.3 Pre/Post Simulation Question 3

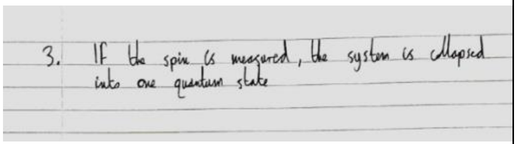
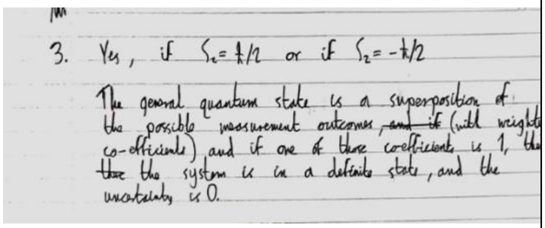
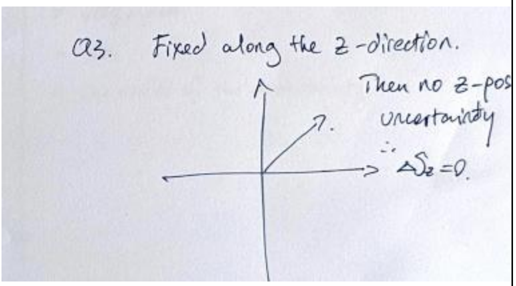
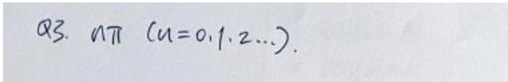
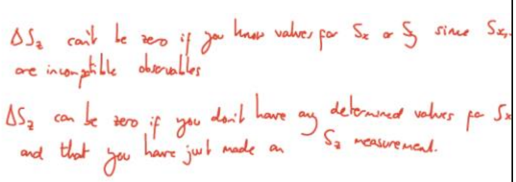
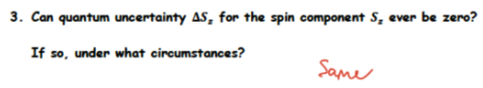
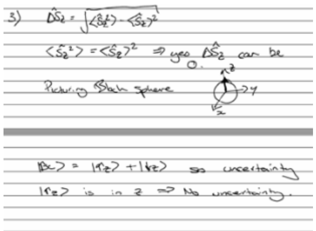
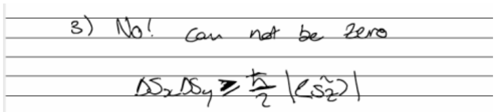
Image 5.17 shows the Pre/Post Simulation Question 3with the correct reasoning. Table 5.3 shows the students pre and post simulation answers side by side.

3. Can quantum uncertainty ΔS_z for the spin component S_z ever be zero? If so, under what circumstances?

Correct Reasoning

- **It can as when a quantum state has only one possible eigenvalue, there is zero quantum uncertainty of what state will be observed when a measurement of the state has taken place.**

[Image 5.17]

Student[12]	Pre-Simulation Answers	Post Simulation Answers
1	 [Image 5.12.a]	 [Image 5.12b]
2	 [Image 5.13a]	 [Image 5.13a]
3	'The Uncertainty being zero when state was in its eigenstate'[13]	Same Answer
4	 [Image 5.14a]	 [Image 5.14b]
5	 [Image 5.15a]	 [Image 5.15b]

[Table 5.3]

12 The Students aren't in the same order as shown in Table 5.1 and 5.2.
13 Due to change in interview technique, answer would spoke out loud.

Unlike the first pre/post simulation question there was no significant change in student's answers, but the hesitancy was non-existence, and they felt a lot more confident in their answers.

6. Discussion

To conclude, the simulation was effective at making student feel a lot more confident in their answers as although for the pre simulation questions 2 and 3 as they used the simulation to create patterns themselves rather than resorting to remembering mathematical formulas which limited their understanding. As there was an inconsistency regarding pre- simulation question 1 in which not all the students were confident in drawing the expectation value and the quantum uncertainty. To rectify, this a challenge 5 was edited to include a question that was centred around the placement of the expectation value and the quantum uncertainty on the histogram. However, due to time constraints of the interviews, many of the students did not have time to go through the challenges which could be an explanation on the reason behind inconsistency.

We were reluctant to include the expectation value on the outcome distribution as we didn't want students to try simply copy this and not venture any further into developing their understanding of quantum uncertainty. This kind of thinking was underlining most of the students answers during the interviews such as thinking in terms of right and wrong answers. I would be very much interested to delve further into how physics can avoid this type of thinking.

7. Acknowledgments

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8. References

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2. Student reasoning about sources of experimental measurement uncertainty in quantum versus classical mechanics
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3. Student interpretations of uncertainty in classical and quantum mechanics experiments
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4. Wave-particle duality and uncertainty principle: Phenomenographic categories of description of tertiary physics students' depictions
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