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Exotic Matter:

An Investigation into Exotic States of Matter and their Properties under the Strong Interaction Using Lattice QCD

Background

Exotic states of matter are states which do not fit into the traditional quark model, in which quarks are confined in bound states called hadrons. Quarks interact under the strong force which is carried by guage bosons called gluons. In this model, quarks can combine to form mesons, dominated by a quark-antiquark pair $q\bar{q}$ or baryons, dominated by three quarks qqq or three antiquarks $\bar{q}\bar{q}\bar{q}$. Any state that does not fit into this model can be considered exotic, for example:

tetraquark/ pentaquark states: $qq\bar{q}\bar{q}$ There are several theories for the structure of such states, such as the molecular picture, in which hadrons are bound together by weaker colour neutral residual QCD force to form molecules, or the hadroquarkonium picture in which the lighter quarks form a quantum mechanical cloud around a heavier quark core.

hybrid meson: $q\bar{q}g$ The gluons in these mesons are non-trivial, i.e the gluonic field is excited and therefore contributes to the valence structure of the particle.

glueballs: These states contain no valence quarks at all and are instead made entirely from interacting gluons.

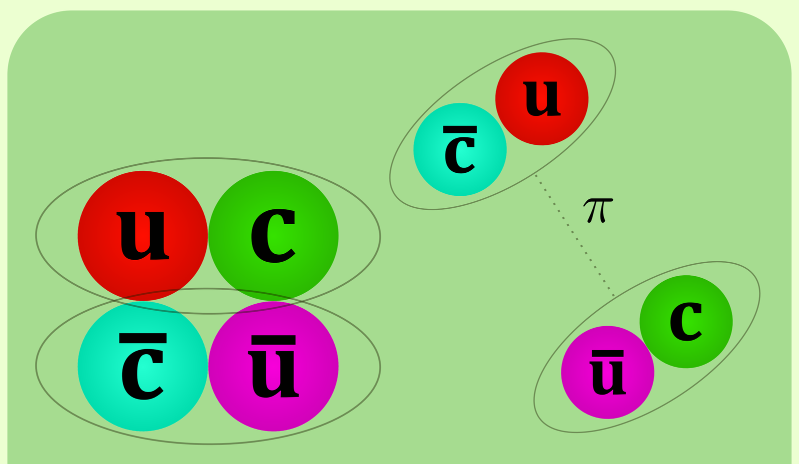


Fig 1:Proposed structures for tetraquark : a) diquark-diantiquark model and b) deuteronium 'molecule'

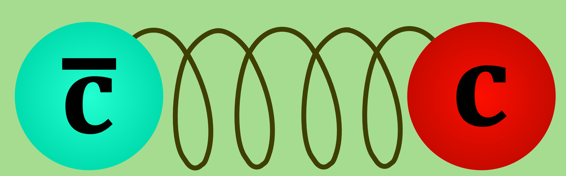


Fig 2: Illustration of a hybrid meson with excited gluonic component

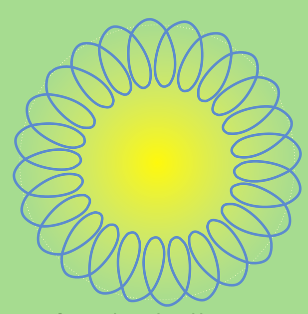


Fig 3: Illustration of a glueball containing just gluons and no valence quarks

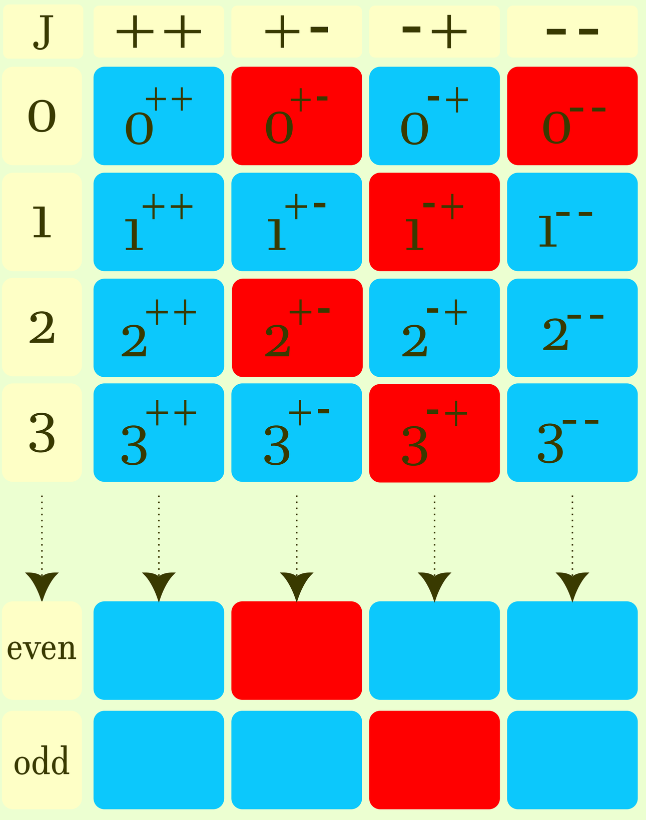


Fig 4: Chart illustrating the pattern seen in quantum numbers where numbers in blue are conventional mesons which fit the traditional quark model, and numbers in red indicate that the state is exotic.

One of the simplest ways to determine whether a state is exotic or conventional is by determining its J^{PC} quantum numbers, where J is total angular momentum, P is parity, and C is charge conjugation. They are defined as:

$$J = L \oplus S = |L-S|, |L-S+1|, ..., |L+S|$$

$$P = -1^{L+1}$$

$$C = -1^{L+S}$$

where L is angular momentum and S is spin.

For conventional $q\bar{q}$ mesons, we know that the spin S will either be 0 or 1, since they are either consist of a quark and an antiquark with spin $\frac{1}{2}$ in opposite directions (S=0) or of a quark and an antiquark with spin $\frac{1}{2}$ in the same direction (S=1). This means that there are certain J^{PC} values which are not 'allowed' under the traditional quark model, namely the case 0^{--} and any even J with a positive P and negative C (J^{+-}_{even}) or any odd J with negative P and positive C (J^{-+}_{odd}).

Lattice QCD (Quantum Chromodynamics) is a method of numerical simulation which can be used to make predictions about interactions of quarks under the strong force. The Lagrangian for Lattice QCD is:

$$L_{QCD} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \sum_f \bar{\Psi}_f(i\gamma^\mu D_\mu - m_f)\Psi_f \quad (1)$$

where f is flavour, a is lattice separation. The covariant derivative D_μ is defined as

$$D_\mu = \delta_\mu - ig(\frac{1}{2}\lambda^a)A^a_\mu \quad (2)$$

This leaves just two input parameters, the coupling g and the bare mass m_f. QCD is a quantum field theory, which means that physical observables $\mathcal{O}$ (or properties that can be measured) can be evaluated as an expectation value over the relevant degrees of freedom.

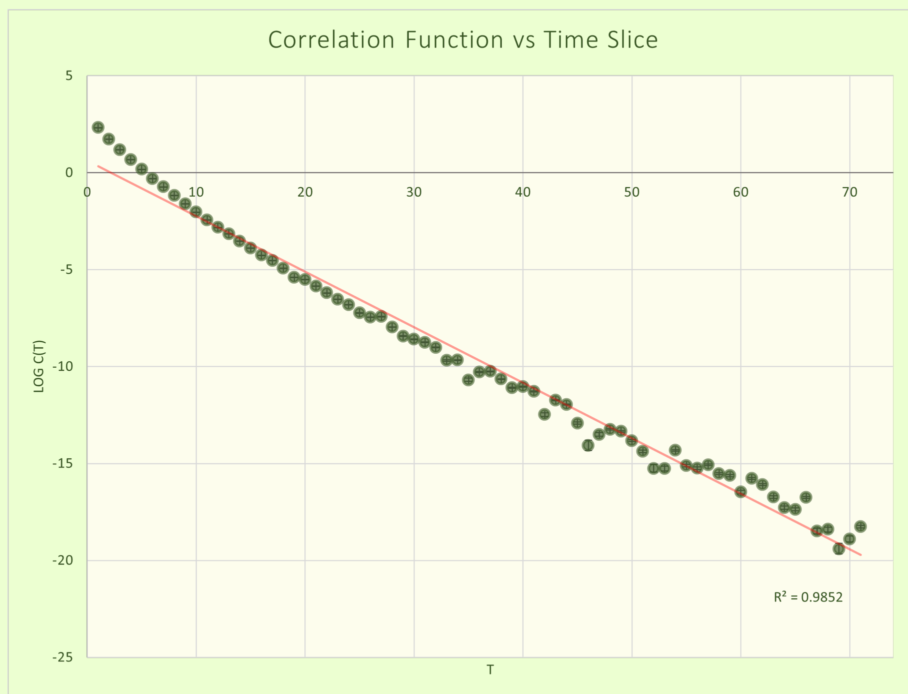
$$\langle\mathcal{O}\rangle = \frac{1}{Z}\int \mathcal{D}U\, \mathcal{D}\Psi\, \mathcal{D}\bar{\Psi}\, \mathcal{O}\, e^{-S_{QCD}} \quad (3)$$

The corresponding two point correlation functions can be written as:

$$C(t) = \sum_{n=0}^{\infty} \frac{|\langle\phi|n\rangle|^2}{2m_n} e^{-E_n t} \quad (4)$$

Discussion

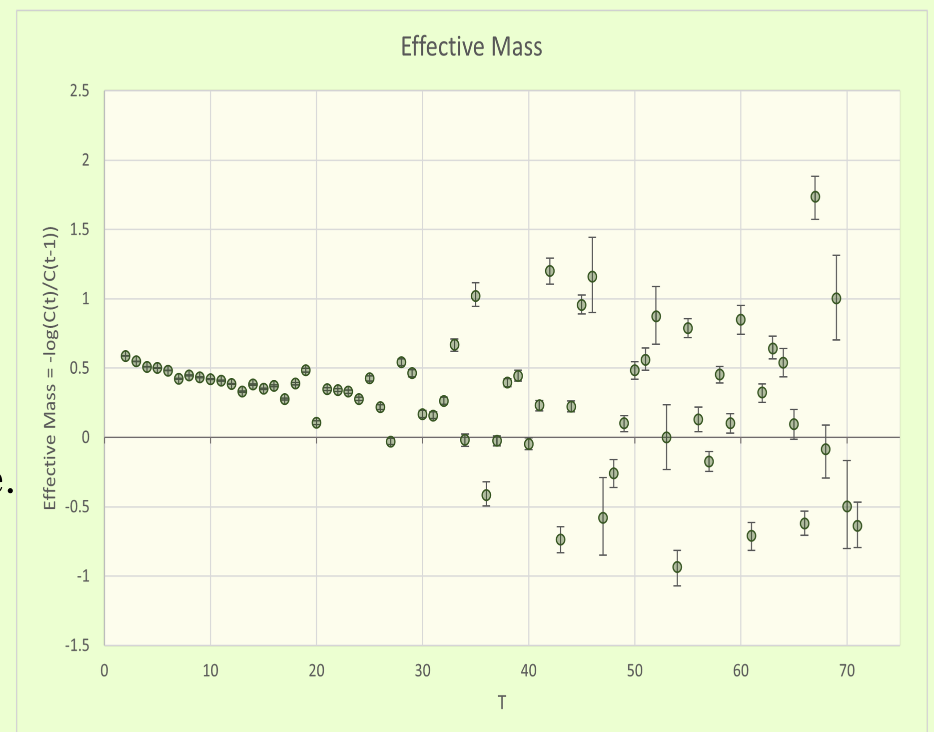
Using a sample set of data for a simulation carried out for an exotic meson, the average correlator was graphed against time slice to obtain the graph opposite.



log(C(t)) was graphed against τ. As expected from equation (4), a linear relationship was found with an R^2 value of 0.9852.

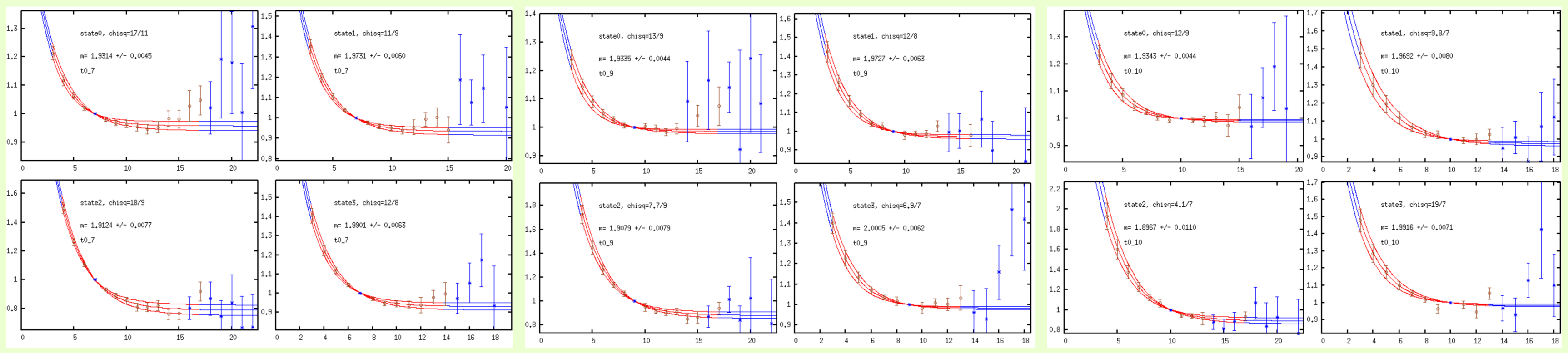
The ground state energy can be estimated from the data using the following relationship, derived from equation (4).

$$a_t m_{\text{effective}} = -\log\left(\frac{C(t)}{C(t-1)}\right)$$

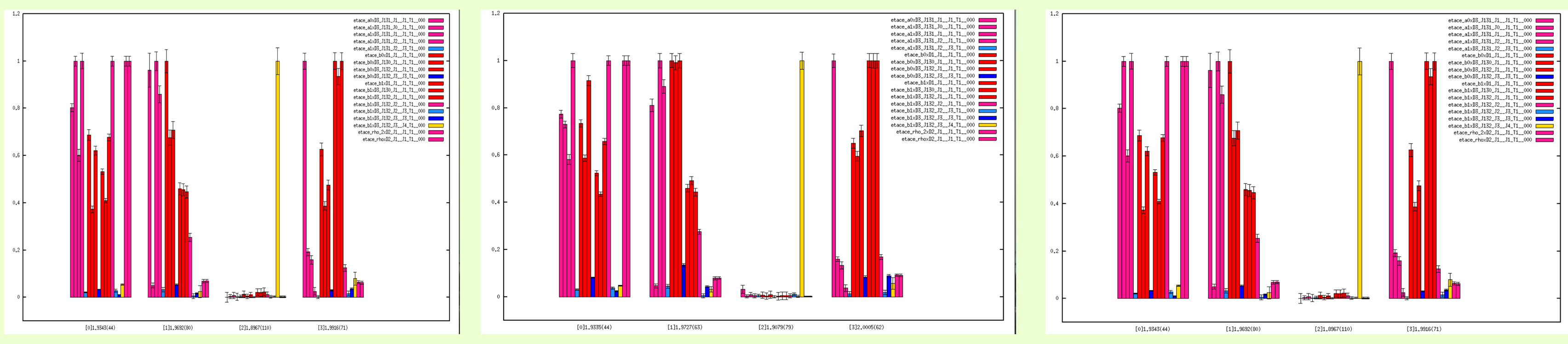


This graph illustrates the relationship between effective mass and time slice. One would expect the estimated effective mass to become constant with increasing time as at higher values of t only the ground state of the system should remain. However this is evidently not the case with the graph above. The sample data used showed a lot of evidence of noise for higher values of t and so a reliable figure for effective mass cannot be found. By refining our choice of operator, our lattice simulation data can be greatly improved.

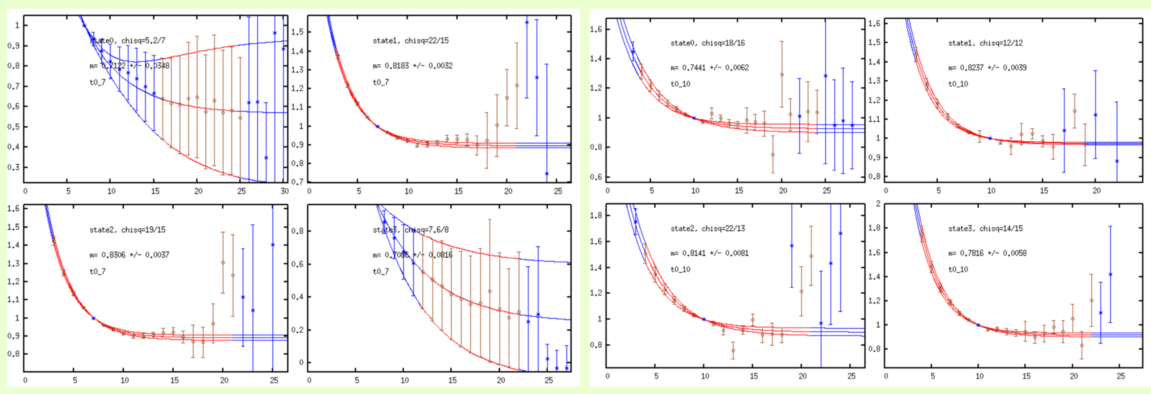
Using an algorithm which uses numerous operators rather than just one, the following graphs were obtained for a 1^{-+} bottomonium exotic hybrid meson. They illustrate how well the generated data overlaps with the proposed operators, with the chi-squared value for each of the first four states for various time slices shown below.



Histogram graphs were also generated which show the overlap between the data and the operators used. The graphs below illustrate this for the same time slices and states as above.



This same process was repeated for the 1^{-+} charmonium exotic hybrid to obtain the graphs below.



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