PROXY WARS: REPEATED BARGAINING BETWEEN PRINCIPALS IN PRINCIPAL-AGENT RELATIONSHIPS

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Abstract. Proxy wars occur when principal states provide military support to agents in civil wars, in order to advance particular political gains. Given the rise of internationalised conflict, it is common for the same principals to engage in multiple proxy wars involving different agents. Such interaction between principals is modelled as a two-level sequential bargaining game with asymmetric information, where each round represents a new proposed political settlement in any of these proxy wars. The model provides a framework to demonstrate the mechanisms by which principals learn and make decisions over time, which determine conflict outcomes, and how the inclusion of agents distorts these mechanisms. Testing of the model against empirical conflict data preliminarily suggests that escalation of a proxy war is linked to prior escalation in related conflicts, and that the under- or over-performance of an agent may have an impact on the outcome of related proxy wars.

1. INTRODUCTION

Proxy wars are an increasingly prevalent form of state conflict with significant political implications. These are conflicts fought between actors known as agents, which are provided military support such as arms, funds, or intelligence by principals. Agents can be state or non-state entities: governments, armed groups, or terrorist organisations. Principals are commonly states, indirectly involved in the conflict in pursuit of their own political objectives.

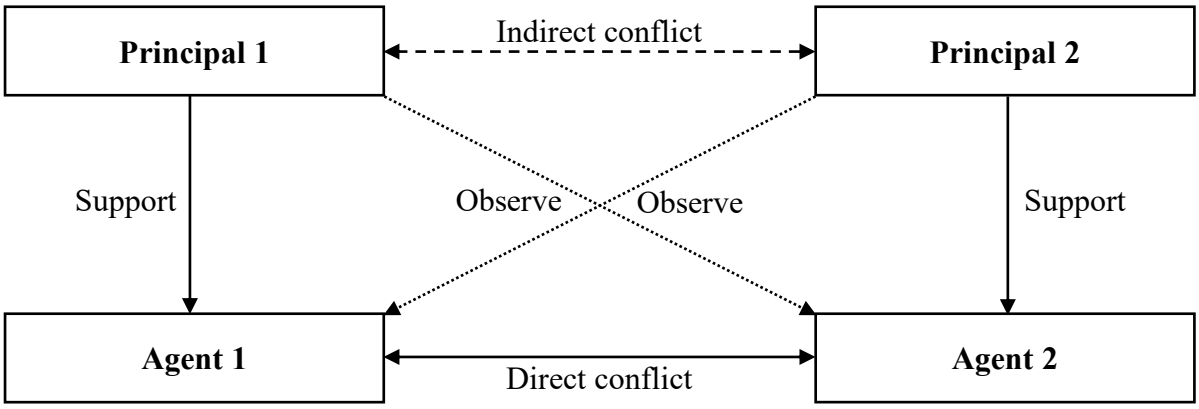

Fig. 1: Interactions between actors in a proxy war.

Despite claims of reduced internal conflict in the post-Cold War period, a rise in the rate of onset of civil war has been observed (Anderson, 2019). In recent years, the growing number of global armed conflicts has largely been driven by internationalised intrastate wars (Davies et al., 2022), where foreign entities intervene in intrastate hostilities, turning civil wars into larger scale conflicts. Proxy wars are defined as such because conflict occurs directly between agents within the same state, but the involvement of other states in the role of principals introduces an internationalised aspect.

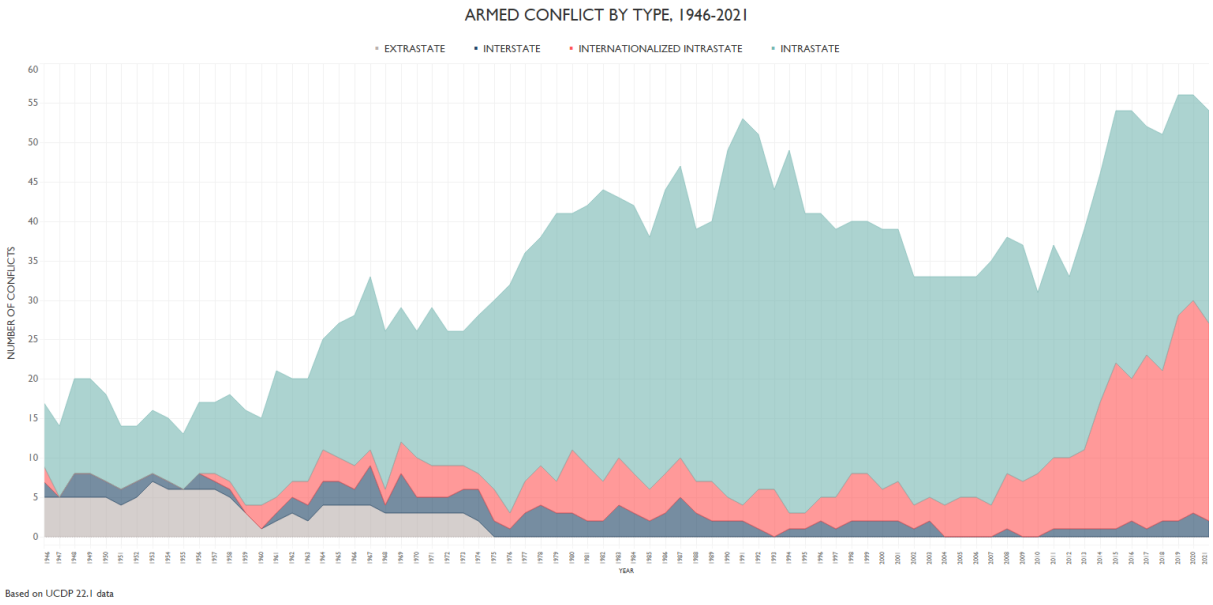

Fig. 2: Trends in armed conflicts by type, 1946-2021. (Davies et al., 2022)

Given that the involvement of multiple state and non-state entities making the implications of proxy war more widespread than that of traditional internal conflict, there is a pressing need to more analytically understand the factors contributing to this rise in proxy war and what outcomes can be expected from them.

With several states historically heavily involved in providing external support in civil conflicts, it is inevitable that they will engage with the same other principal states multiple times. The Cold War era saw the United States and the Soviet Union on opposing sides of different proxy wars; today, the United States and Russia continue to be major players in internationalised conflict, backing both state and non-state actors in civil wars in Syria, Ukraine, and beyond. Such repeated interaction creates scope for reputation-building between states, with both sides developing an increasingly informed understanding of each other's military strength. This

implies that decisions made in one proxy war have an impact on decisions made in concurrent or future proxy wars fought between the same principals. Escalation in severity of any one proxy war raises concerns of similar escalation or prolonged endurance of related conflicts, given the potential for complex interconnectedness between them. The involvement of agents adds an additional level to the relationship: how agents represent the strength of their principals influences their principals' reputations, and therefore also the outcomes of proxy wars.

This study models proxy war as a sequential bargaining game to examine the relationship between escalation in one proxy war and the outcomes of other proxy wars fought between the same principals. The addition of a second level to the game, representative of the involvement of agents, allows for analysis of how the principal-agent relationship affects the outcome of this bargaining process.

2. LITERATURE

2.1 What do we know about proxy wars?

Proxy wars have three potential outcomes: escalation, de-escalation, and endurance. Escalation is normally signalled by an increase in intensity of the conflict; this is often measured using empirical proxies such as the number of battle deaths. De-escalation occurs when principals come to a settlement, ending the conflict. Until either of these happen, when proxy wars continue with negligible change to their severity, they are said to endure.

Among the three, the causes of endurance have been the most widely discussed. Cost-benefit analysis by principals and agents is one of the prevailing explanations for this: each actor weighs the payoffs gained in a proxy war against what they stand to lose from engaging in it and decides whether to protract the conflict or take action to end it (Powell, 2019). Other explanations include aligned but distinct political goals of principals and agents (Elias, 2022), adaptation of policies to shifts in the political landscape (Sottimano, 2019), and rates of change of power distribution (Powell, 2012). It has also been suggested that balancing the need to support their agent with the avoidance of conflict escalation may distort the aid that principals provide, preventing both defeat of their agent and a decisive victory (Anderson, 2019).

In general, conflict can be modelled as sequential bargaining between two actors, with each offer or decision constituting a single bargaining round. Proxy wars, though independent in

themselves, can also be seen as consecutive rounds of a bigger bargaining game or conflict, spanning multiple interactions of the same two principal actors. One critical property of the game is asymmetric information: conflict literature has established that fighting only occurs in the absence of complete information, or there would be no uncertainty affecting bargaining offers that creates the risk of rejection and war (Powell, 1987). Another is its sequential nature, reflecting repeated interaction (Morrow, 1989).

These shape the interactions and learning of the bargaining actors, providing structure for further analysis of the bargaining process. The strength of a state is made known to its opponents by its reputation, shaped by beliefs held about that state; with asymmetry of information, reputations may be accurate or inaccurate reflections of true bargaining power. Between states that interact repeatedly, however, this uncertainty must reduce as each state learns more about the other over time. This raises the question: much like how decisions within a single conflict affect concurrent or subsequent decisions, do related proxy wars have an effect on each other, and can this explain the outcomes of each? Given the added complexity of the second layer of interaction between agents, it is worth considering how the principal-agent relationship may distort this process.

2.2 What do we not know about proxy wars?

It would be natural to begin by asking to what extent these wars are interconnected, and by which principal states. Knowing this would aid in further analysis of the impact of one proxy war on another's outcome. If, for instance, Russia escalated internal conflict in Ukraine by direct involvement, would this alter the course of action taken against it by the United States in the civil war in Yemen? The commonality of principal actors that learn and adapt over time suggests that this is possible. Since there has been significant discussion on what may cause proxy wars to protract but less on why they end or escalate, focusing on how escalation of one proxy war might cause escalation, de-escalation, or endurance of related proxy wars not only explores all outcomes, but immediately addresses the implications of the most severe of the three.

In determining the effect of the principal-agent relationship on bargaining, much of existing literature focuses on how the cross-level relationship between principal and agent entities leads to the respective outcomes of conflict (Elias, 2022; Konyukhovskiy & Grigoriadis, 2019;

Sottimano, 2019)[1]. Understanding the mechanisms by which these outcomes come about, however, first requires an understanding of the base level bargaining between principals. Unlike earlier crisis bargaining models, this must define fighting between principals as an inside option rather than one that ends the bargaining process — otherwise, it would be impossible to model escalation of one conflict while others are ongoing. Once it is determined how principals learn and come to decisions at each round of the game, the second level of representation of principals by agents can then be added. Separating the analysis by level allows for identification of the exact points at which the basic bargaining process is distorted, further shedding light on the mechanisms behind the outcomes of proxy wars.

3. MODEL

3.1 Satisfaction in the bargaining game

Before modelling proxy wars as a bargaining game, it is necessary to understand why states engage in bargaining. States bargain when at least one among them is dissatisfied with the division of a shared whole. Whether a state is satisfied with its share depends both on the status quo division and the cost of fighting the other state to gain a greater portion of the whole.

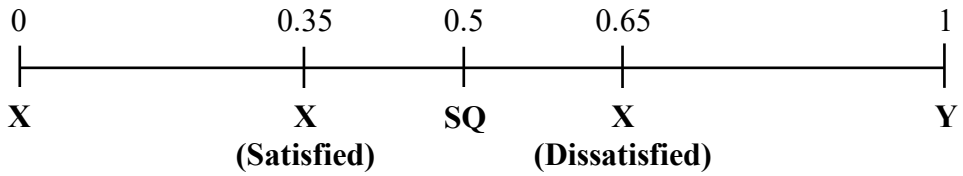

Fig. 3: Satisfied and dissatisfied states in bargaining.

The bargaining occurs between two states, *X* and *Y*. The status quo division, denoted by SQ, divides the whole $\pi = 1$ equally between *X* and *Y*.

Consider the bargaining process from the perspective of *X*. *X*'s utility of accepting the status quo is $U(SQ) = 0.5$. To understand why *X* would be satisfied or dissatisfied, the utility of accepting the status quo must be compared with the utility of fighting *Y* for a larger share.

[1] These focus on case studies of selected proxy wars, explaining their protraction with greater focus on the principal-agent relationship, rather than the conflict relationship between principals.

If *X* fights *Y*, let its probability of winning be $P = 0.75$. *X*'s utility of fighting is $U(f) = P^{\pi} - C$, where C is the cost to *X* of fighting. Since $\pi = 1$, this simplifies to $U(f) = P - C$.

First let the cost of fighting for *X* be high, $C_H = 0.4$. This results in the utility of fighting being less than the utility of accepting the status quo:

$$U(f) = P - C_H = 0.75 - 0.4 = 0.35 < 0.5$$

If the cost of fighting for *X* is low, $C_L = 0.1$, *X* gains more utility from fighting than accepting the status quo:

$$U(f) = P - C_L = 0.75 - 0.1 = 0.65 > 0.5$$

When $U(f) < U(SQ)$, as in the first instance, *X* is satisfied and prefers to accept the status quo; but when $U(f) > U(SQ)$, *X*, being dissatisfied, prefers to fight *Y*. This begins the bargaining game.

3.2 One-level bargaining: Bargaining between principals

Bargaining on the principals' level can be modelled using the costly-process model by Powell (2004), which is a sequential, recurring game with asymmetric information. Two states are in conflict over the division of a shared whole, representing political control over a territory. One state, *S*, is satisfied with the status quo division, while the other, *D*, is dissatisfied with it.

Each round begins with an offer by *S* of how the shared whole might be divided. Let this offer be $x \in [0, 1]$, where x is the fraction of the whole offered by *S* to *D*. *D* can now respond in three possible ways: accept x, reject x, or fight.

Accepting x ends the game with the settlement $(1 - x, x)$ accorded to *S* and *D* respectively. Rejecting x gives *S* the choice of continuing the game by starting a new round, or fighting *D*. Regardless of whether *S* or *D* decides to go to war, fighting incurs costs s for *S* and d for *D*, with the probability $k_S \in [0, 1]$ that *S* collapses and the probability $k_D \in [0, 1]$ that *D* collapses; together, this gives rise to the probability $k \equiv 1 - (1 - k_S)(1 - k_D)$ that one or both states collapse, ending the game. If one state collapses, the other gains the entirety of the shared whole. If both states collapse, they fall back to the status quo division. Otherwise, if both prevail, then the game continues, and a new round begins.

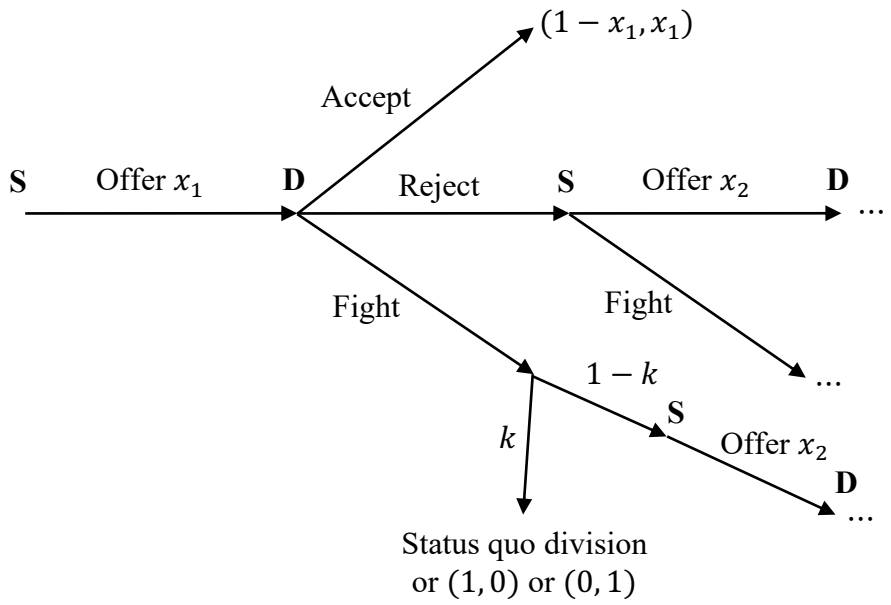

Fig. 4: Bargaining game between principals.

In the specific context of proxy wars, *S* and *D* are the same two principal actors engaged in multiple different proxy wars over time. Every new round represents a new proposed settlement of political control in any of these proxy wars, moving in chronological order. Take, for instance, Iran and Saudi Arabia, providing external support to opposing sides of civil conflict in both Syria and Yemen. A decision by Saudi Arabia to provide funding in Yemen constitutes a "proposition" to Iran, marking one round. If this is followed chronologically by Iran's provision of logistical support in Syria, two rounds have passed. The third round may return to the conflict in Yemen when either principal makes another proposition.

Acceptance of a settlement ends the proxy war, rejection without fighting causes it to endure, and fighting represents escalation. Empirically, the collapse of a state results in the same outcome as acceptance of a settlement.

Each state's cost of fighting and probability of collapse determine its bargaining power in the game. If *S* was certain about *D*'s cost of fighting (d) and its probability of collapse (k_D), it would offer *D* the corresponding x in the first round of the game, eliminating the risk of rejection or fighting. Therefore, for bargaining to be necessary, *S* must lack information about d and k_D, leading to uncertainty about *D*'s bargaining power and what x must be offered for *D* to accept. *S* also believes, as Powell assumes, that *D* prefers to fight to the finish over accepting the status quo division.

3.3 One-level bargaining: Screening between principals

d and k_D define *D*'s bargaining power, or its type. Stronger states with more bargaining power have lower costs of fighting and lower probabilities of collapse, while weaker states with less bargaining power have higher costs of fighting and higher probabilities of collapse. From Powell's results, strong dissatisfied states fight more rounds and obtain greater shares in their settlement. Weak dissatisfied states are quicker to accept settlements, in which they are offered a smaller share.

With more information about *D*'s type, *S* can make offers that are closer to *D*'s acceptable offer given its true bargaining power, which lowers the risk of costly conflict. *S* therefore has incentive to screen *D* in order to determine its type, as this informs the offer *x* it makes at every round of bargaining.

The screening process begins with *S* making an initial offer. *D* can respond by accepting, rejecting, or fighting *S*. Say *D* does not accept the offer. If *D* fights, there is a probability $(1 - k)$ of neither state collapsing, upon which *S* restarts the bargaining process with a new offer. If *D* rejects, *S* can choose to fight, with the same probability of collapse as above, or to immediately restart the new round of bargaining. *S* updates its beliefs about *D*'s type from observation of *D*'s behaviour — the process by which *D* builds its reputation with *S*. Over multiple rounds, this reduces the information asymmetry that affects the offers *S* makes to *D*.

S learns through two different mechanisms. One is dependent on *D*'s rejection of *S*'s offer, and the other on *D*'s choice to fight *S*. If *D* rejects *S*'s offer, then *S* updates its beliefs through the first mechanism only. If *D* fights *S*, then *S* updates its beliefs through both mechanisms, since a rejection of *S*'s offer is implicit in the choice to fight.

<u>Rejection Mechanism</u>

To see how *S* learns from *D*'s rejection of its offer, let r be the current round of the game between *S* and *D*. Over rounds 1, 2, …, r, if *D* neither collapses nor accepts *S*'s offer, *S* makes offers $x_1 < x_2 < \cdots < x_r$ with increasing concession. These offers correspond to what Powell refers to as "cut points" in terms of d and k that reveal information to *S* about *D*'s type.

In terms of cost of fighting, *S*'s offers correspond to cut points $d_1 > d_2 > \cdots > d_r$, where d_r is the cost of fighting that makes *D* indifferent between acceptance of *S*'s r-th offer and fighting the r-th round before accepting *S*'s $(r+1)$-st offer. The series of cut points decreases in cost of fighting: the lower *D*'s cost of fighting, the higher the offer needed to make it indifferent between accepting the current or next offer, since fighting an additional round could enable it to gain greater concession from *S*. By rejecting *S*'s r-th offer, *D*'s behaviour communicates to *S* that its cost of fighting has the upper bound d_r (or else it would have accepted a previous offer).

Similarly, *S*'s offers correspond to cut points $k_1 > k_2 > \cdots > k_r$, where k_r is the probability of collapse that makes *D* indifferent between acceptance of *S*'s r-th offer and fighting the r-th round before accepting *S*'s $(r+1)$-st offer. The lower *D*'s probability of collapse, the higher the offer needed to make it indifferent between accepting the current or next offer, reflected by the cut points decreasing in probability of collapse. *D*'s rejection of *S*'s r-th offer communicates to *S* that its probability of collapse has the upper bound k_r (or else it would have accepted a previous offer).

At every round, *S* updates its beliefs about the upper bounds of d and k, which informs subsequent offers made.

Fighting Mechanism

Let k_D be the probability of collapse of a strong state and k'_D be the probability of collapse of a weak state, where $k_D < k'_D$, since a weak state is more likely to collapse. *S* believes that the probability distribution of *D*'s collapse is Θ, with density θ. Therefore, *S* initially believes that the odds that *D* is a strong state versus a weak state is $\theta(k_D)/\theta(k'_D)$.

After *D* engages in one round of fighting and does not collapse, this ratio changes. The probability of *D* being a strong state and having prevailed in that round is $(1-k_D)\theta(k_D)$. The probability of *D* being a weak state and having prevailed in that round is $(1-k'_D)\theta(k'_D)$. Therefore, the odds that *D* is a strong state versus a weak state now becomes $(1-k_D)\theta(k_D)/(1-k'_D)\theta(k'_D)$, or the odds before fighting multiplied by a factor of $(1-k_D)/(1-k'_D)$. Recalling that $k_D < k'_D$ gives $(1-k_D)/(1-k'_D) > 1$, so *S* now believes that there is a greater likelihood that *D* is a strong state instead of a weak one.

The probability of *D* being a strong state rather than a weak one becomes higher with every round that *D* prevails. The change in the probability ratio is the mechanism through which *S*'s beliefs about *D*'s type is updated at every round of the game. *S*'s offers change accordingly, with greater concessions made as the probability of *D* being a strong state increases.

3.4 Two-level bargaining: The principal-agent relationship

Incorporating the principal-agent relationship into the bargaining model does not require a change in the basic structure of the game, but primarily affects the screening mechanism between principals. Recall that *S* modifies its beliefs about *D*'s bargaining power through observation of *D*'s behaviour. With the involvement of agents, however, *S* observes the behaviour of *D*'s agent rather than *D* itself. In the Angolan Civil War, the Soviet Union would have drawn conclusions about the bargaining power of the United States indirectly from the actions of its agent, UNITA; similarly, the United States would have observed the Angolan government to gain information about the Soviet Union's strength.

If the agent's behaviour does not accurately reflect *D*'s bargaining power, *D*'s reputation with *S* diverges from its true type. Because this informs offers made to *D*, introducing this second layer to the game affects the outcome of principals' bargaining.

Agents can be compared based on how their inclusion affects their principal state's reputation. Underperforming agents reflect only a part of their principals' true bargaining power; overperforming agents make their principals appear stronger than their true type; and perfect agents are accurate reflections of their principals. Because the type of agent determines the reputation of its principal, it should have an impact on the outcome of bargaining rounds between principals.

Empirically, however, it is difficult to determine the true bargaining power versus perceived bargaining power of a state. Instead, the definitions of different types of agents can be modified to compare agents' performance under the same principal. Between consecutive rounds of a bargaining game, an agent underperforms if it reflects a weaker bargaining power than a previous agent, and overperforms if it reflects a stronger bargaining power than a previous

agent. Consistent with the previous definition, the effects of different types of agents are reflected in the outcome of principals' bargaining.[2]

3.5 Propositions and hypotheses

To determine how the escalation of one proxy war has an impact on other wars between the same principals, consider how this is modelled by Powell's bargaining game. Escalation in this model refers to an increase in intensity of fighting between principals across all the proxy wars that they are engaged in, rather than any one specific war. If either state collapses, the other state wins the game. If *D* prevails in round n, then *S* updates *D*'s reputation as a stronger state via both the Rejection and Fighting mechanisms. This should result in a greater concession to *D* between rounds n to $n+1$ as compared to rounds $n-1$ to n, when *D* only rejected *S*'s offer and neither state chose to fight.

Proposition 1. If escalation occurs in round n, and *D* prevails, the ratio x_{n+1}/x_n is greater than the ratio x_n/x_{n-1}.

Since it is difficult to obtain data on concessions made by one state to another, the observable effects of a greater concession by *S* to *D* must be considered instead. Empirically, escalation in round n may be represented by an increase in battle deaths from round $n-1$ to round n of the game, or an increase in the number of ongoing proxy wars fought between the same two principals from round $n-1$ to round n; here, both are used.

Assuming a greater concession signals to *D* that *S* now considers it a stronger state, *D* would predict that *S* increases its offer in the following round by more than before and is less likely to choose fighting over making another offer. As every round of this raises x to a higher value, *D* has incentive to escalate subsequent rounds with the aim of continuing to build its reputation, earning it a greater share once the conflict terminates. Therefore, if escalation occurs in the

[2] Sawyer et al. (2017) have discussed how the degree of fungibility of external support affects outcomes of civil conflicts, where highly fungible external support results in greater uncertainty about how accurately the principal's bargaining power is represented by its agent. Leveraging on these results, fungibility of external support can be used as a predictor of performance of an agent: the higher the fungibility of support, the greater the disparity may be between a principal's true bargaining power and what is represented by their agent. This method of approximating an agent's performance to determine its type can be explored in future iterations.

overall conflict between principals, individual proxy wars should see an increase in intensity of fighting in the following round.

Hypothesis 1. If the cumulative number of battle deaths in related proxy wars increases from year $n-2$ to year $n-1$, and the number of ongoing conflicts between principals increases from year $n-2$ to year $n-1$, then the number of battle deaths in a proxy war increases from year $n-1$ to year n.[3]

A change in the type of agent between rounds of bargaining also affects concessions made to *D*. Assume *D*'s type of agent is changing, while *S*'s type of agent remains the same, since *D* is the state building a reputation in this model. If the agent in round $n+1$ underperforms compared to the agent in round n, *S* updates its belief of *D* being a stronger state, but to a lesser extent than between the two previous rounds. Therefore, the concession by *S* between rounds n to $n+1$ is smaller than between rounds $n-1$ to n.

Proposition 2. If *D*'s agent in round $n+1$ underperforms compared to the agent in round n, the ratio x_{n+1}/x_n is smaller than the ratio x_n/x_{n-1}.

Underperformance of an agent can be reflected by a decline in the number of battle deaths between two consecutive years. To distinguish this from de-escalation of the conflict as a whole, de-escalation is now represented by a reduction in the total number of proxy wars ongoing between the principals.

The smaller concession signals to *D* that its reputation as a strong state with *S* has improved by a smaller degree than before. In response, it is likely that *D* will either provide greater support to their agent in the following round or escalate the following round, in hopes of exhibiting greater bargaining power. Assuming the agent continues to reflect the same proportion of the now increased support as in the previous round, both responses should result in an increase in intensity of the following round.

[3] Although *D* is more likely to escalate related proxy wars, *S* may be less likely to escalate these conflicts, considering *D*'s updated reputation as a stronger state. However, since current data makes it empirically challenging to identify which role each state plays in bargaining, it is only possible to observe the aggregated impact of escalation on related conflicts, not the individual decisions of states.

Confirmation of Hypothesis 1 would suggest that the effect of *D*'s escalation is stronger than the effect of *S*'s reluctance to escalate, which in turn suggests that the change in ratio referenced in Proposition 1 is significant.

Hypothesis 2. If the cumulative number of battle deaths in related proxy wars declines from year $n-2$ to year $n-1$, and the number of ongoing conflicts between principals does not decline from year $n-2$ to year $n-1$, then the number of battle deaths in a proxy war increases from year $n-1$ to year n.

Conversely, if D's agent overperforms in round $n+1$ compared to round n, the concession by *S* between rounds n to $n+1$ should be greater than between rounds $n-1$ to n.

Proposition 3. If *D*'s agent in round $n+1$ overperforms compared to the agent in round n, the ratio x_{n+1}/x_n is greater than the ratio x_n/x_{n-1}.

D has no incentive to increase costly support for its agent when the agent is building a better reputation for *D* than it would have built for itself. It is therefore probable if *D*'s agent is overperforming that *D* will not escalate related proxy wars; neither will it de-escalate them, as it aims to obtain the maximum offer from *S*. It is challenging to identify an empirical hypothesis to represent Proposition 3, but testing of Hypothesis 2 will suffice to confirm that the type of agent affects the outcomes of related proxy wars.

4. EMPIRICS

4.1 Data

Data on proxy wars was obtained from the Uppsala Conflict Data Programme (UCDP) database. The External Support Dataset provided information on principals and agents engaged in a conflict, which aided in identifying principal-agent and principal-principal dyads. As identification codes of state and non-state actors were differentiated (state actors had IDs below 1000, non-state actors had IDs above 1000), this allowed for identification of only principal-agent dyads where principals were states and agents were non-state entities. Best estimates of battle deaths from the Battle-Related Deaths Dataset were collated for each proxy war, with data reported on an annual basis.

All data was compiled and analysed for empirical testing using R.

4.2 Empirical testing

Preliminary analysis of actors and conflicts

To determine if the outcomes of conflicts between the same principals have an effect on each other, it is first necessary to visualise how principal actors are related, and how proxy wars are

related. Doing so entails understanding which conflicts are shared between the same principals. This can be visualised as a map of nodes to represent the connections between conflicts.

Doing so entails a record of each principal-agent pair that occurs in the database and the proxy war they are involved in. For this, data on the principal-agent pair involved in each conflict was obtained from the External Support database: the Principal ID, the corresponding Agent ID, and the Conflict ID. The data was first filtered for unique entries and non-state principals were eliminated. Data was then compared row-by-row iteratively (Row 1 vs Row 2, Row 1 vs Row 3, …, Row 2 vs Row 3, Row 2 vs Row 4…).

Principal ID	**Agent ID**	**Conflict ID**
365	700	137
670	1135	137
⋮	⋮	⋮

Table 1: Example of conflict data obtained from the External Support database.

To create Table 2, rows from Table 1 were compared iteratively. A data entry was made in Table 2 if the two rows shared the same Conflict ID and one agent was a state entity, while the other was a non-state entity (one agent had an ID above 1000 while the other had an ID below 1000). This ensured that the principals were on opposing sides. The principal dyad was created for each entry by combining the IDs of the two principals, with the smaller ID number ordered first. The example data entry in Table 2 corresponds to the two rows of data in Table 1.

Conflict ID	**Principal 1**	**Principal 2**	**Principal Dyad ID**
137	365	670	365-670
⋮	⋮	⋮	⋮

Table 2: Example of data table to obtain mapping of conflicts by common principal dyads.

A symmetric square matrix was then created from Table 2. Each row or column corresponded to a unique Conflict ID; therefore, every entry (i, j) was equal to the number of principal dyads shared between the conflicts represented by row i and column j. In the example matrix, $(1, 2)$ and $(2, 1)$ both represent the number of principal dyads in common between conflicts 137 and 191. The diagonal was again set to zero to avoid comparing a conflict to itself.

$$\begin{array}{c} \text{Conflict ID} \\ 137 \\ 191 \\ 192 \\ \vdots \end{array} \begin{array}{c} \begin{array}{cccc} 137 & 191 & 192 & \cdots \end{array} \\ \begin{bmatrix} 0 & 1 & 2 & \cdots \\ 1 & 0 & 1 & \cdots \\ 2 & 1 & 0 & \cdots \\ \vdots & \vdots & \vdots & 0 \end{bmatrix} \end{array}$$

Fig. 5: Example of symmetric square matrix with corresponding unique conflict IDs.

Two conflicts were considered to be linked if they shared the same principal dyad. Comparing the rows of Table 2 iteratively, if two conflicts shared the same Principal Dyad ID, the matrix entry corresponding to the relevant conflicts would have its count increased by one. This matrix was then converted into a mapping of conflict nodes.

Empirical testing of hypotheses

Hypothesis 1 and Hypothesis 2 were tested using linear regression modelling. Table 3 summarises the explanatory variables and regressors for Models 1 and 2, corresponding to their respective hypotheses.

Change in number of battle deaths in proxy war from year $n-1$ to year n	$change\_bd$
Number of battle deaths in proxy war in year n	bd_best
Change in cumulative battle deaths in proxy wars between same principals from years $n-2$ to $n-1$	$change\_bd\_sp$
Change in number of ongoing conflicts between same principals from years $n-2$ to $n-1$	$change\_on\_sp$
Interaction variable (product of $change\_bd\_sp$ and $change\_on\_sp$)	$change\_bd\_sp * change\_on\_sp$

Table 3: Explanatory variables and regressors in linear regression modelling.

Model 1:

$$change\_bd = \beta_0 + \beta_1 change\_bd\_sp + \beta_2 change\_on\_sp + \beta_4 change\_bd\_sp * change\_on\_sp$$

The explanatory variable was the change in battle deaths in a given proxy war from year $n-1$ to year n, with years as a proxy measurement for rounds. Regressors were the change in

cumulative battle deaths in related conflicts from years $n-2$ to $n-1$, the change in number of ongoing conflicts between the principals from years $n-2$ to $n-1$, and an interaction variable of the previous two regressors. The interaction variable represents the effect on $change\_bd$ associated with a change in $change\_bd\_sp$, dependent on the change in $change\_on\_sp$. This is equivalent to the same effect associated with $change\_on\_sp$, dependent on $change\_bd\_sp$.

Model 2:

$$bd\_best = \beta_0 + \beta_1 change\_bd\_sp + \beta_2 change\_on\_sp + \beta_4 change\_bd\_sp * change\_on\_sp$$

The explanatory variable was the number of battle deaths in a given proxy war in year n, with the same regressors as the first model.

4.3 Results

The map of conflicts linked by common principal dyads is shown in Figure 6.

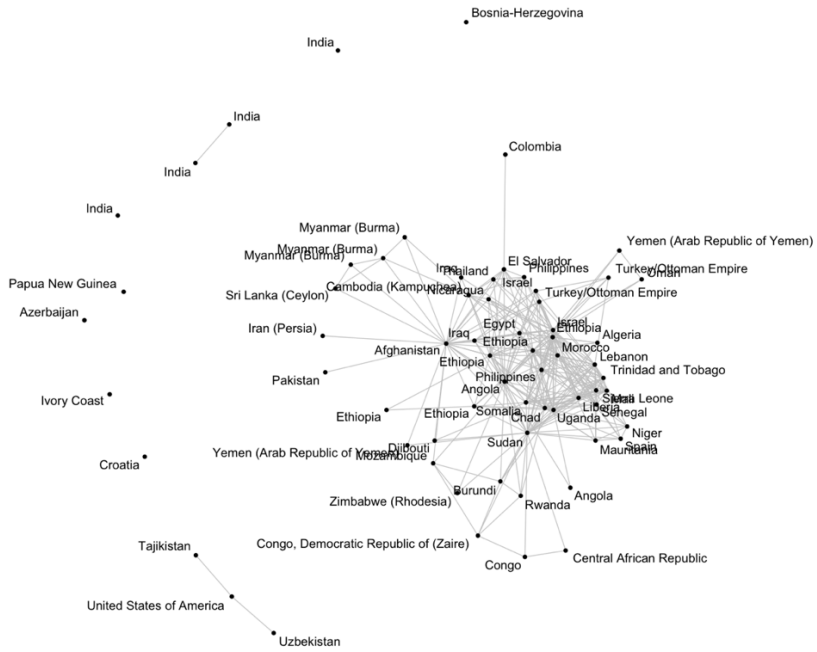

Fig. 6: Mapping of conflicts with common principal dyads.

Figure 6, in operationalising the linkages between armed conflicts, aids in visualising the extent of connectivity between proxy wars and the principals involved in them. This is a preliminary step in investigating how the connectivity may contribute to interdependence between the

outcomes of these conflicts. This network is also used as a spatial weight matrix in the empirical models. By multiplying the spatial weight matrix with the dependent variable, a spatial lag is attained, which we temporal lag as well to address simultaneity concerns. A variable is then created which indicates the number of other conflicts proxies involved in a particular conflict are involved in.

In testing hypothesis 1, we find some support that when the number of battle deaths in related proxy wars increases from year $n-2$ to year $n-1$, and the number of ongoing conflicts between principals increases from year $n-2$ to year $n-1$, then the number of battle deaths in a proxy war increases from year $n-1$ to year n. This interaction is plotted in the left panel of Figure 7, where the number of battle deaths in related conflicts is only positive at higher values of conflict the principals are involved in. The interaction almost reaches standard levels of significance. This suggests that the escalation of one conflict may result in escalation in related conflicts, as one state predicts a greater concession of the other and seeks to maximise its gains by taking advantage of this.

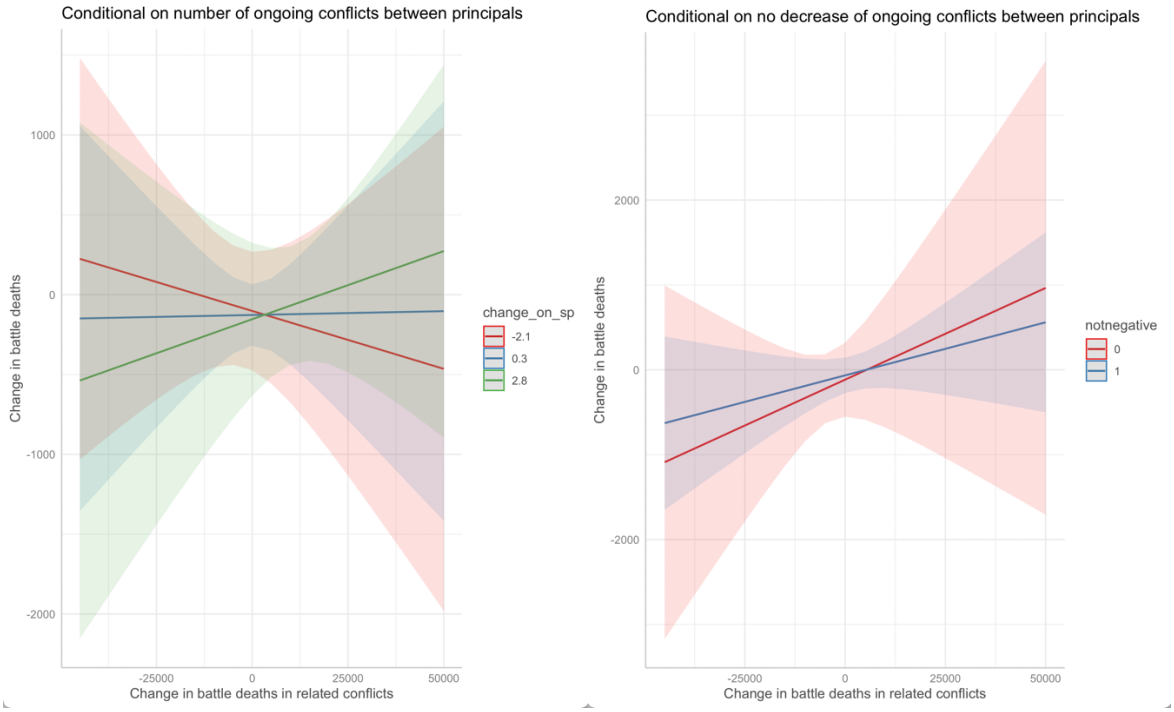

Fig. 7: Modelling of hypotheses 1 (left) and 2 (right), showing the relationship between the change in battle deaths in a given conflict and the change in battle deaths in related conflicts between the same principals.

In testing hypothesis 2, we find less support that if the cumulative number of battle deaths in related proxy wars declines from year $n-2$ to year $n-1$, and the number of ongoing conflicts between principals does not decline from year $n-2$ to year $n-1$, then the number of battle deaths increases in year n. This interaction is plotted in the right panel of Figure 7, where the relationship between a change in battle deaths in related conflicts and battle deaths is always positive and not significantly moderated by whether there is no decline in conflicts two principals are involved in. Therefore, there is currently insufficient support for the hypothesis that the performance of an agent in one conflict has an impact on the outcome of related proxy wars.

5. DISCUSSION

There remain several points to be addressed in future work. The bargaining process has been modelled as a single offer made per round by the satisfied principal: Powell (2004) expands upon this original model to incorporate multiple offers per round, to determine the effect of the bargaining environment on its outcome. It is also unrealistic in practice to limit the ability to make bargaining offers to one principal alone. However, the simplified model has been retained for the current analysis, holding the bargaining environment constant, to facilitate study of the relationships between outcomes of the bargaining instead.

Empirically, it is challenging to differentiate between "rounds" of a bargaining process. If defined as decisions made in conflicts, these rounds are likely to occur simultaneously, or be so closely consecutive that it is difficult to differentiate their effects. In this analysis, a round is measured by proxy as a calendar year, but this measurement has scope to be improved upon.

A final consideration is whether explicitly modelling for bluffing by principal affects the results of the analysis. By behaving like a stronger state than its true bargaining power defines it as, a principal runs a greater risk of collapse, but may be able to obtain a larger settlement. The question is whether such a principal would make the same decisions of escalation or endurance as predicted in the hypotheses, or if the additional risk would push it to "cut its losses" and end the bargaining game.

6. CONCLUSION

The nature of bargaining in proxy wars as repeated interaction with asymmetric information raises the question of the interconnectedness of these conflicts' outcomes. Study of the principal-agent relationship explains how the second bargaining level distorts the process of the first, contributing a further layer of analysis to the above. Together, these provide a holistic approach to understanding both levels of bargaining within the same bargaining process by anchoring the analysis to the base-level bargaining between principals. This contrasts with current literature, which focuses on cross-level relationships or bargaining between agents to explain outcomes of proxy wars.

To understand the base level of bargaining, the study began with a theoretical model of proxy war, adopted from a conflict model by Powell (2004) designed to represent bargaining and reputation-building with war as an inside option. The model was then expanded upon to incorporate principal-agent interaction, with explicit focus on how the second level of bargaining alters the process of reputation building.

The predicted outcomes of bargaining and learning gave rise to theoretical propositions, from which hypotheses were derived for empirical testing. This was conducted using R with data from the UCDP database.

The current results suggest that connections between the outcomes of proxy wars are worth further analysing, especially with the considerations of more realistic modelling and data that is more representative as a proxy of theoretical variables. The rise in internationalised conflict and growing prominence of proxy war emphasises the need for this. Understanding the mechanisms of proxy wars is crucial to predicting their outcomes, which is in turn necessary in providing institutions that will mitigate the inevitable costs of war.

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