



Introduction:

Geometric symmetry is a familiar concept to most people who have been taught maths to a primary school level, but symmetry – the property by which a system is unchanged under a transformation – is a mathematical concept that extends beyond geometry. Symmetry is important in physics – it allows physical systems to be formulated in the most simple way possible, deepening our understanding of them. Symmetries can be difficult to identify but once identified can open the door to large break-throughs (for example, the 2008 Nobel Prize in Physics was awarded to a group who made a break-through regarding symmetry and particle physics).

Recently, a MIT research group^[1] successfully trained a Neural Network to identify symmetries in classical and quantum mechanics problems, but as their method could only search for symmetries from a prespecified list, they suggested further work to make symmetry generators trainable. This suggestion had already been accomplished for a certain class of symmetries in 2019^[2]. Their algorithm was restricted to symmetries of the ‘symplectic’ kind and depended on the solutions of the system’s equations of motion being solvable. This condition is not a problem when considering many physical systems, but it means that the method cannot be applied to understand systems such as the three-body problem which cannot be solved (solved meaning the system’s behaviour being predicted at any time).

The algorithmic application of abstract algebra to identify Lie symmetries has also been utilised, but this approach is also limited to systems whose equations of motion can be solved.

In this project, I will build a novel symmetry-identifying algorithm which doesn’t require an inputted list of prespecified symmetries or solvable equations of motion since it will be constructed on the one idea that is fundamental to symmetry – change.

Methodology and Timeline:

This project will focus on simple classical mechanical systems and train an algorithm to search for an infinitesimal spatial transformation under which the mechanical system satisfies Noether’s Theorem (a theorem that allows a symmetry and a corresponding conserved quantity to be identified from the effect of a change on the system’s Lagrangian). This method is a suitable choice for early attempts to train an algorithm to generate symmetries since it is the initial approach taught to students in identifying symmetries.

A benefit of the proposed approach is that it allows symmetries to be identified in systems whose equations of motion cannot be solved, which is not true of the research mentioned in the introduction to this proposal^{[1][2]}. Furthermore, since infinitesimal translations/rotations/combinations of both are the fundamental ideas from which the symmetries



emerge, the proposed method can theoretically identify the same classes of symmetry as those identified by previous researchers whose algorithms were confined to working on integrable systems only^{[1][2]}. The proposed approach is an innovation in this sense, having the same capabilities as past research to identify symmetries, but being less confined in terms of what mechanical systems it can work with.

The first two weeks of the research project will be dedicated to:

- Identifying the environment that will be most useful to me in building the proposed algorithm (I have experience in C++, Python and MatLab, so I will consider libraries that I can utilise with these languages in particular – I will probably use a Python or MatLab environment to avoid unnecessary time loss when coding).
- Fully defining the workflow of my algorithm, the loss function that the algorithm will be learning to minimize, how my algorithm will identify total time derivatives and how I will parameterize the transformations that the algorithm will have freedom to define to find symmetries. (A reinforcement learning algorithm is the type of machine learning algorithm that would be applicable in the workflow in this project, so I intend to use an Artificial Neural Network (ANN) in this project.)

The next three weeks will be dedicated to coding and testing the proposed symmetry identification algorithm. I will achieve this by first building an algorithm that works for the simplest case – so that it is able to generate a linear translation and identify when a system is homogenous in space. Once this simplest case algorithm is working correctly, I will add to the algorithm to give it the extra functionality of being able to generate any combination of translations and rotations. This method of building a working algorithm for the simplest case, then working to expand its capabilities has proven successful for me during my coding experience, allowing me to separate the complexities in the algorithm from the complexities in the systems it is applied to and to work on the complexities from the two different sources separately.

The final week will be dedicated to interpreting the results of the research project, assessing the level success attained by the project and presenting my findings in poster and report form.

Intended Outcomes:

This project's intended outcome is the creation of a functional algorithm that is capable of identifying symmetries in classical mechanical systems that the algorithm itself has generated from transformations comprising of the fundamental changes that can be made to a classical system. The success of the algorithm will thus be measured by investigating its ability to identify known symmetries in classical mechanical systems, ranging in complexity from homogeneity in space (a



symmetry whose corresponding conserved quantity is momentum) to the symmetry that corresponds to the conserved quantity known as the Runge Lenz vector in the Kepler problem.

This project's success in identifying known symmetries in classical mechanical systems would indicate that the process of the proposed algorithm could be applied to more complex systems to identify symmetries and deepen our understanding of them.

References:

- 1: Liu, Z., & Tegmark, M. (2022). Machine Learning Hidden Symmetries. *Physical Review Letters*, 128(18), 180201. <https://doi-org.elib.tcd.ie/10.1103/PhysRevLett.128.180201>
- 2: Bondesan, R., & Lamacraft, A. (2019). *Learning Symmetries of Classical Integrable Systems*. International Conference on Machine Learning, Los Angeles, California, United States of America. <https://doi.org/10.48550/arXiv.1906.04645>