

Inverse Problems and International Competition

Claire Hannick, supervised by Professor Daniel Lesnic

Introduction

The research focuses on inverse problems, where the objective is to deduce unknown causes from observed effects, a crucial area in mathematics with wide-ranging applications. These problems are particularly relevant in fields such as medical imaging, geophysics, and engineering, where direct observation is not feasible. Through this research, I explored the mathematical techniques behind inverse problems and participated in international competitions to disseminate and engage with global research communities.

Aims and Objectives

- To understand and explore the mathematical techniques used to solve inverse problems.
- To examine real-world applications of inverse problems, particularly in medical imaging, seismology, and non-destructive testing.
- To participate in academic competitions and conferences to present findings and foster collaborative learning.
- To develop research, problem-solving, and communication skills applicable to future academic and professional settings.

Research Methodology

- **Literature Review:** Conducted a thorough review of mathematical methods used to solve inverse problems, including regularization techniques and Bayesian inference.
- **Collaborative Discussions:** Engaged with peers and faculty at the International Mathematics Competition (IMC), discussing research findings and exploring practical applications.
- **Problem-Solving Approach:** Worked on theoretical and applied inverse problems, leveraging mathematical models and computational tools to gain insights into real-world scenarios.
- **Dissemination:** Presented research through posters and discussions, receiving feedback from international mathematicians.

Results

- **CT and MRI Imaging** are examples of inverse problems. In CT scans, the goal is to reconstruct a 3D image of the internal structures of the body from 2D X-ray projections. The processes use inverse algorithms like the Radon transform or Fourier inversion to reconstruct the images.
- **Earthquake Location:** The epicenter and depth of earthquakes can be inferred by solving inverse problems, where you work backward from the times seismic waves reach sensors to pinpoint where and when the quake occurred.
- **Stellar Structure Inference:** By observing the light spectra emitted from stars, astronomers solve inverse problems to infer the composition, temperature, and internal structure of stars.

Conclusion

The research into inverse problems has highlighted their significant practical applications across multiple fields. In medical imaging, inverse problem-solving techniques are essential for reconstructing internal body images from external data, improving diagnostics. In geophysics, these methods allow scientists to model the Earth's interior using seismic data. The study of inverse problems also supports advancements in engineering, particularly in non-destructive testing. By solving these real-world challenges, inverse problems contribute to technological innovations that benefit society. The research outcomes emphasise the critical role mathematics plays in addressing practical, real-life issues through problem-solving and computational methods.