

Active Matter Control Simulations

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Active Matter Background

- Active matter is a class of non-equilibrium systems where constitutions generate self-propulsion by fuel consumption leading to complex collective phenomena.
- A single active particle trapped in a potential well can be modelled by the following Langevin equation with random thermal noise and propulsion terms.

$$m\ddot{r} = \underbrace{-\gamma r}_{\text{Drag}} - \underbrace{\partial_r \phi(r, \alpha)}_{\text{Interaction}} + \underbrace{\gamma\sqrt{2D}\eta}_{\text{Thermal Noise}} + \underbrace{\gamma v_a}_{\text{Propulsion}}$$

- By imposing an overdamped regime we can solve the equation of motion to attain the particle velocity.

$$\dot{r} = -\mu \partial_r \phi(r, \alpha) + \sqrt{2D}\eta + v_a$$

- By defining an appropriate potential trap and suitable Gaussian processes for noise, we can construct the updating equations required to simulate the trajectory of particles over time.

- To model the activity we use an Ornstein-Uhlenbeck process.

$$\tau \dot{v}_a(t) = -v_a(t) + \sqrt{2D_v}\zeta(t)$$

Potential Trap

- A harmonic potential well traps particle positions in one dimension with variable strength over time.

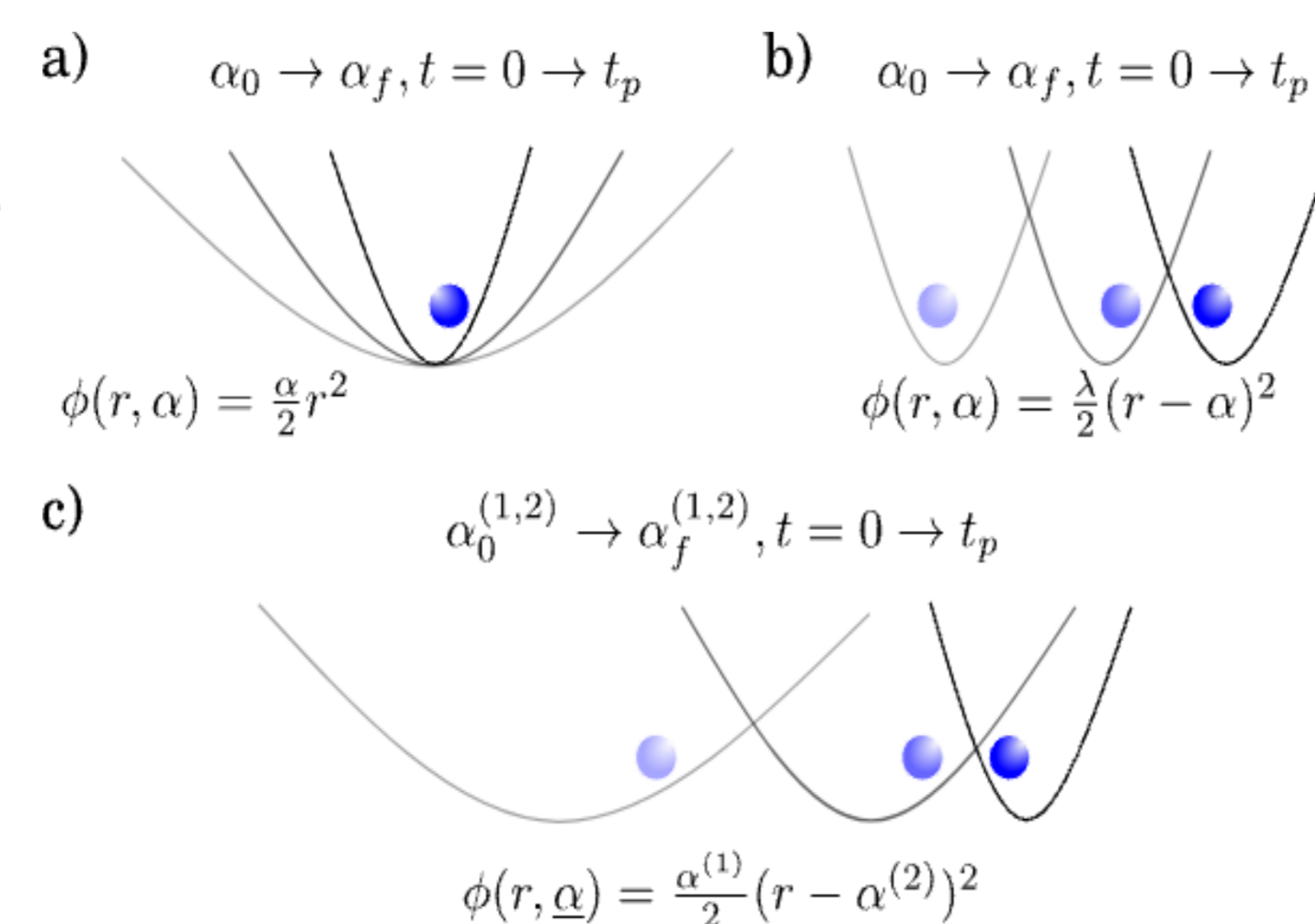
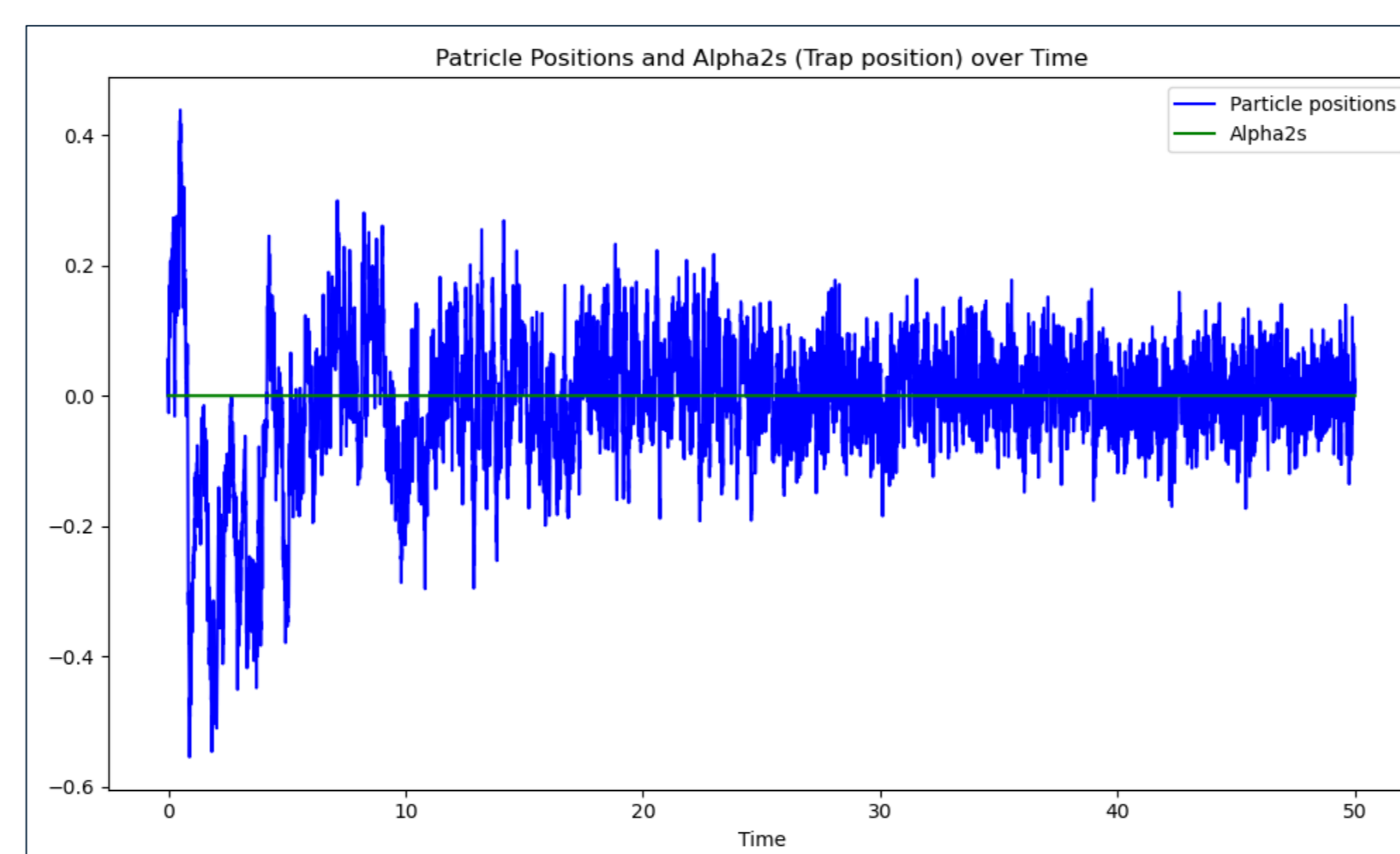


Fig 1: Different trap regimes under varying position and strength

- For control, we consider the following three cases of varying the parameters individually: strength and relative position.

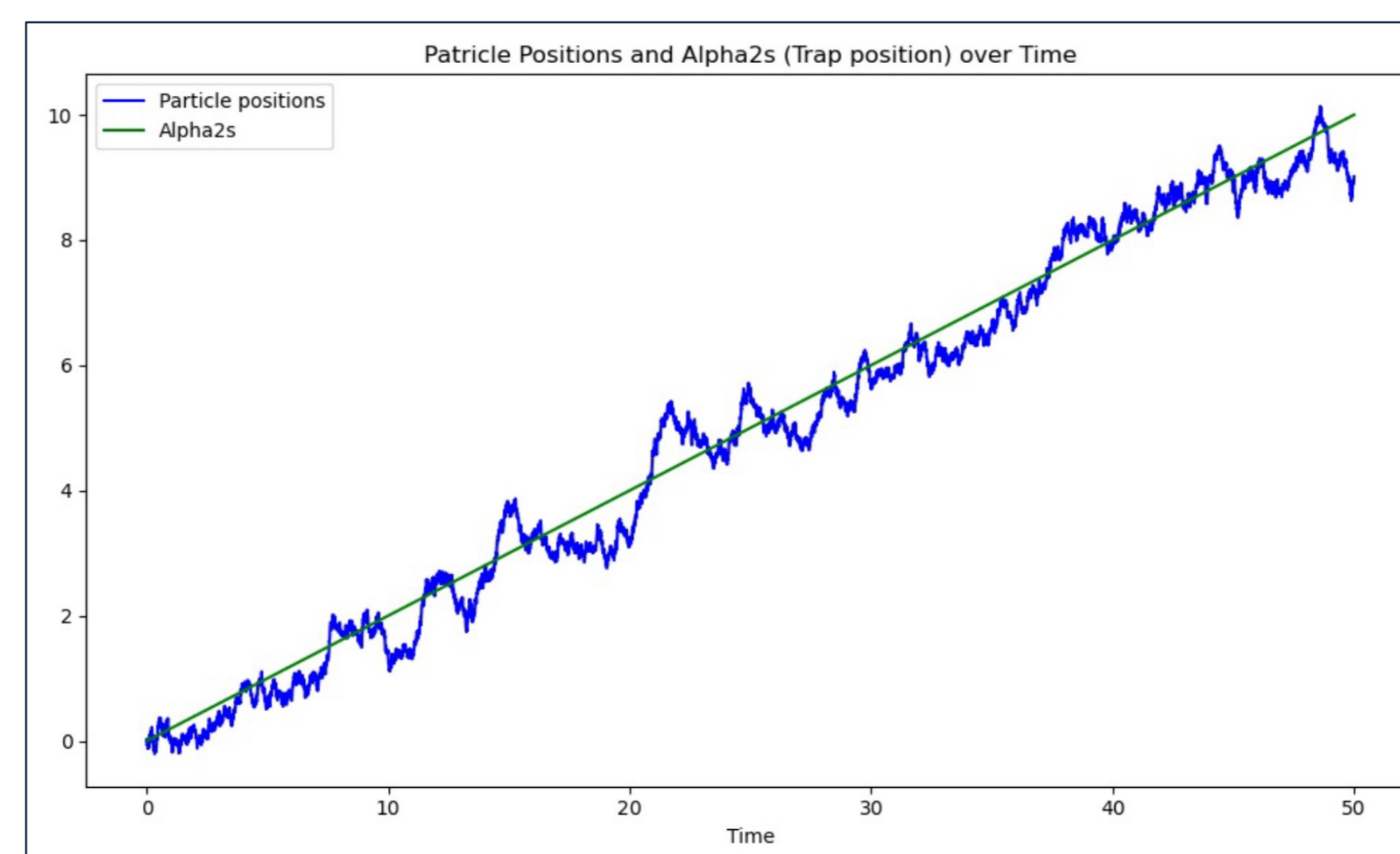
Different Control Regimes

- Control over the active particle is demonstrable as a response to changing trap parameters.
- Under each regime, the particle adapts its motion with some lag in time to the changing trap strength or position.



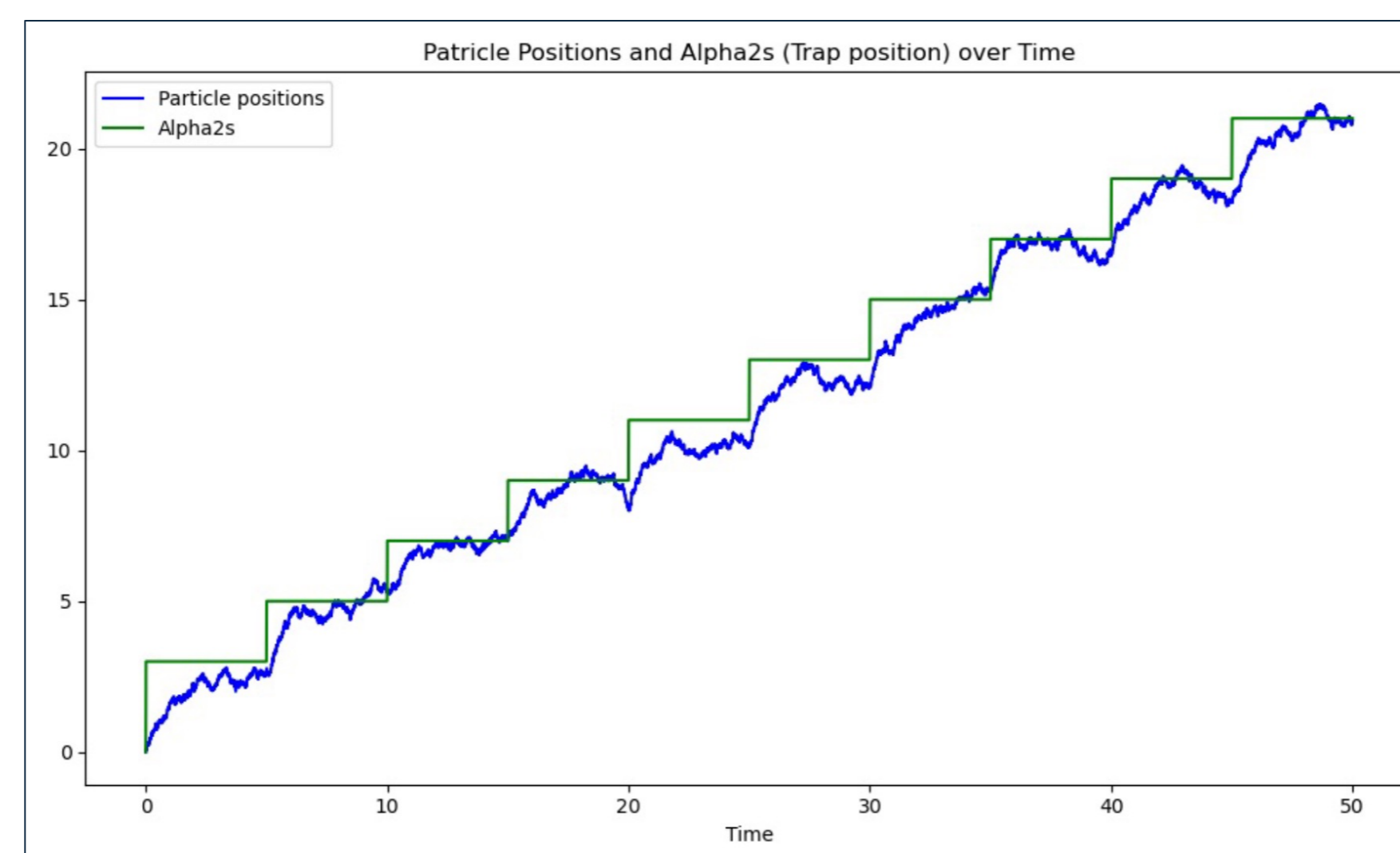
A: Increasing trap strength linearly over time holding position constant

Particle position amplitude is dampened



B: Smooth linear increase in trap position over time at constant strength

Particle follows the trap with some lag



C: Trap Position changed with a step function over time at constant strength

Particle follows the trap with jumps after each step

Fig 2: Particle trajectory across time under different trap regimes A,B,C

Simulation Analysis

- We compare particle trajectories over many simulations and then compute and compare with theoretical expectations of key test statistics under different trap regimes

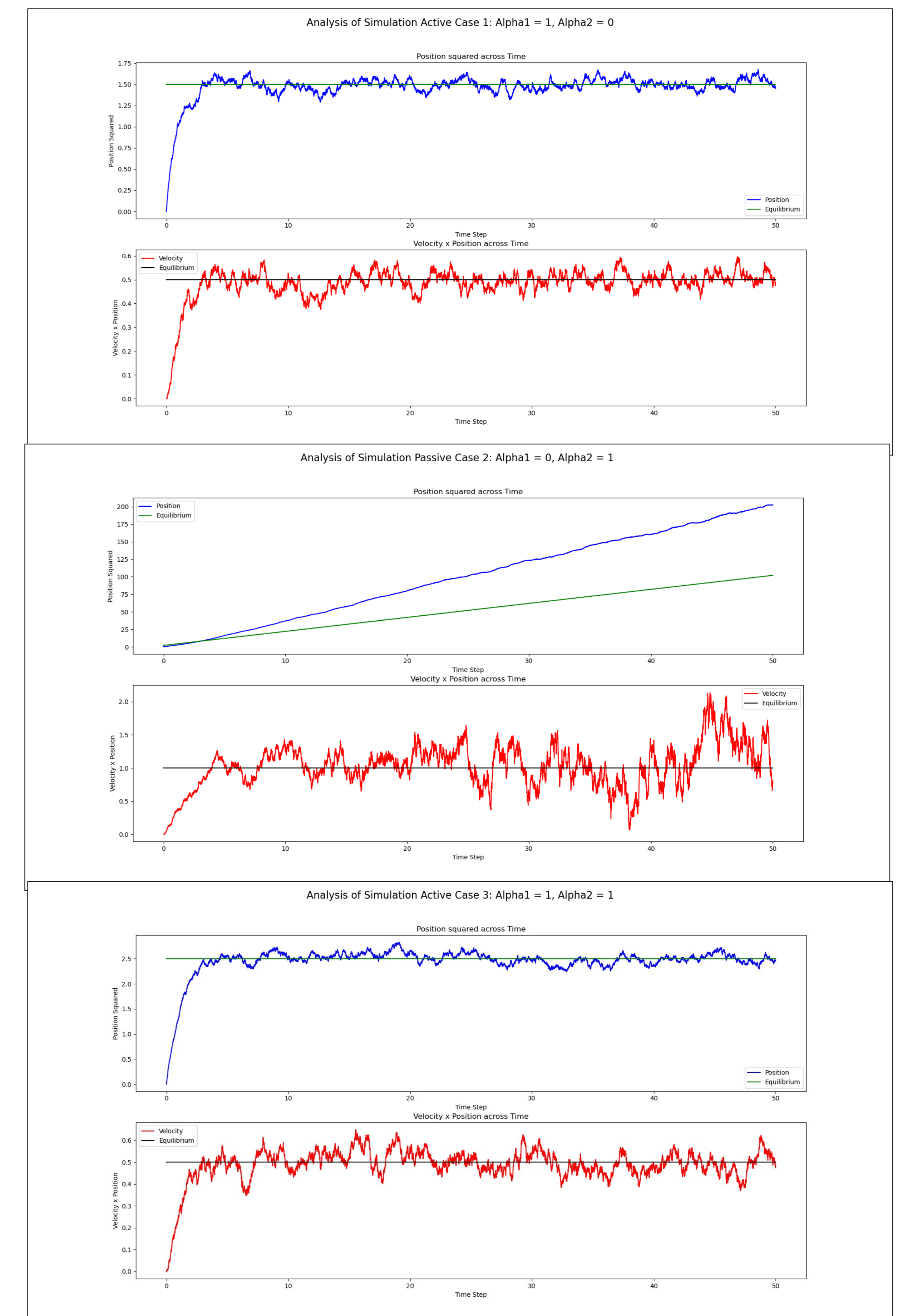


Fig 3: Test statistics in cases a) strong trap, b) no trap strength, c) strong trap with movement