Dynamic analysis & optimization of pump foil

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Outline:

1. Introduction

Figure 1: Pump foil (source: Alpine foil)

Emerging in the early 2000s, pump foiling (Fig. 1) is a relatively new and still poorly understood water sport. It involves the vertical oscillation of a rider on a hydrofoil board, a motion that, somewhat counterintuitively, generates forward propulsion without the aid of wind, waves, or external power sources. This mechanism allows practitioners to glide over water at remarkable speeds. Although the physics behind pump foiling may appear unconventional, the system shares clear analogies with aviation, as it leverages lift forces and fluid dynamics to maintain motion—albeit on a much smaller and human-powered scale.

The scientific interest in pump foiling extends beyond its recreational appeal. Understanding its mechanics can provide insights into unsteady propulsion, energy efficiency in oscillatory systems, and more efficient water locomotion. However, despite these promising connections, research on pump foiling remains scarce, and much of its underlying physics has yet to be systematically modeled.

The objectives of this internship were therefore twofold:

- To develop a new theoretical and computational model for this underexplored system, establishing a framework that can serve as a basis for future studies.
- To determine the optimal set of parameters (specifically pumping frequency and applied force) that maximize forward speed.

2. Developing a working physical model

a. Force equilibrium

The initial reference material provided by the laboratory consisted of a short video recording of a pump foil being used on a Swiss lake. A preliminary analysis of this footage was carried out to gain an initial understanding of the motion. However, the dynamics remained unclear at this stage and required further evaluation. As such, the first task was to construct a schematic representation of the system, identifying the forces acting upon it.

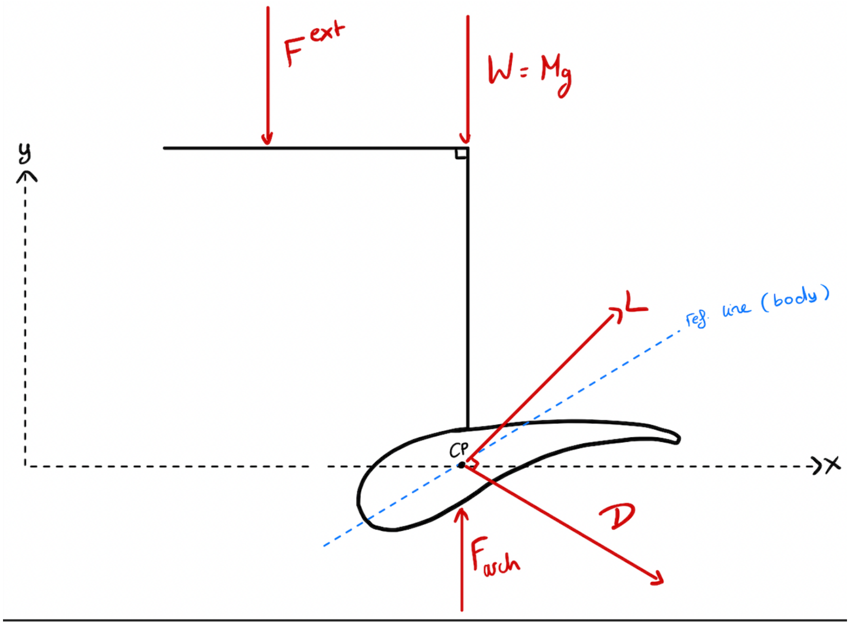

Figure 2: Schematic model of the system + forces

We can see the horizontal line at the top representing the board, with the two points of applications representing a person's feet.

The point labeled CP, for center of pressure, is a unique point in an aerodynamic body where the lift and drag forces are applied.

The forces at play are:

- Weight of the person: The standing person's weight is applied at one of the feet. Naturally, the expression of the weight is $\boldsymbol{W} = M\boldsymbol{g}$.
- External applied force: The other foot, the one furthest from the foil applied an oscillating force we can model as: $\boldsymbol{F^{ext}} = -Fsin(\omega t)\hat{\boldsymbol{y}}$. This force is essential as it is

the reason why the system moves forward, generating lift and forward motion through fluid flows.

- Buoyancy: When submerging a body within a fluid, an upward facing force is generated, proportional to the mass of fluid displaced. Here, this force acts as a 'safety net', preventing the board from sinking if it ever reaches below the waterline. As we will see later, an effective pump foil scarcely uses this force as the board normally never reaches below the waterline. Despite this it is still important to include this force if we want to create an all-encompassing model. The expression for buoyancy is: $\boldsymbol{F_{arch}} = m_{fluid} g \boldsymbol{\hat{y}}$

The remaining two forces are known as **aerodynamic forces**, and rise from the plane-like nature of the foil. They are applied in a point known as the center of pressure of the foil. These forces are also present in planes and other aircraft, with the lift force being the reason why such systems fly.

- Lift force: It is created when air moves faster over the curved top surface of the wing than underneath, causing lower pressure above and higher pressure below. This pressure difference, combined with the wing's angle of attack, generates lift. The expression for this force is much more complex than previously. Indeed, we can write $\boldsymbol{L} = \frac{1}{2}\rho \boldsymbol{U}^2 C_L(\alpha) A$.
  ρ is the density of fluid (in our case, water), U is the velocity of the incoming fluid flow in the reference frame of the wing, CL is a dimensionless coefficient (known as lift coefficient) that depends on the wing's geometry and the angle of the fluid flows relative to the wing (known as the angle of attack), and finally A is the area of the wing.

- Drag force: This force is generated by the resistance an object experiences as it moves through a fluid, like air or water. It acts opposite to the direction of motion and is caused by friction between the surface and the fluid, as well as pressure differences around the object. The faster the object moves or the less streamlined it is, the greater the drag. The expression is analogous to lift, $\boldsymbol{D} = \frac{1}{2}\rho \boldsymbol{U}^2 C_D(\alpha) A$, with a difference in the coefficient, here written as CD, drag coefficient.

b. Differential equations

To understand the ways this system moves, we must use the fundamental law of mechanics, projected on the x and y axis:

$$\sum \boldsymbol{F_x} = m\boldsymbol{a_x}$$
$$\sum \boldsymbol{F_y} = m\boldsymbol{a_y}$$

For convenience, we denote $\ddot{x} = a_x$ and $\dot{x} = v_x$ with the dot operator being a time derivative.

The first 3 forces are only present in the y plane, so they are easy to integrate onto our formula. However, lift and drag have components in both x and y directions, so it is essential we find a way to project them effectively.

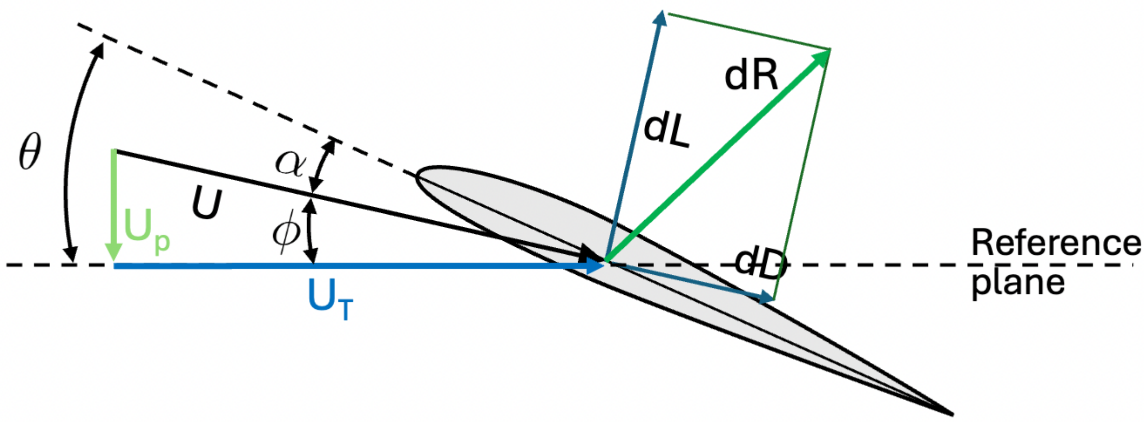

Figure 3: Angles of motion of the foil

We can see (Fig. 3) that projecting the forces in the correct reference plane gives us:

$$\boldsymbol{L} = -Lsin\phi\boldsymbol{\hat{x}} + Lcos\phi\boldsymbol{\hat{y}}$$
$$\boldsymbol{D} = -Dcos\phi\boldsymbol{\hat{x}} - Dsin\phi\boldsymbol{\hat{y}}$$

Therefore, the total projected sum of forces can be written as:

$$m\ddot{x} = -Lsin\phi - Dcos\phi$$

$$m\ddot{y} = Lcos\phi - Dsin\phi - Mg - Fsin(\omega t) + F_{arch}$$

We have several problems at this point:

1. The angle phi is undefined as it changes through time
2. If you recall the expressions of the lift and drag forces, the coefficients CL and CD are functions of the alpha angle (cf. figure 2) which also varies through time.

c. Angles of motion

To counter these problems, we will introduce three angles of motion that we will determine:

- $\boldsymbol{\alpha}$**, alpha: the angle of attack** - Angle created by the incoming velocity and the reference line of the wing.
- $\boldsymbol{\theta}$**, theta: the pitch angle** - Angle created between the reference line of the body and the x plane.
- $\boldsymbol{\phi}$**, phi, the velocity angle** – Angle created between x and y components of the incoming velocity

Trigonometry in the triangle created by the U, U_T, U_P vectors yields:

$$\tan(\phi) = \frac{U_P}{U_T} = \frac{-v_y}{-v_x} = \frac{\dot{y}}{\dot{x}}$$

Which we can invert as:

$$\phi(t) = \arctan\left(\frac{\dot{y}}{\dot{x}}\right) \qquad (1)$$

Also, we can clearly see that, geometrically:

$$\theta = \alpha + \phi \qquad (2)$$

Hence we only need to find either $\theta(t)$ or $\alpha(t)$ in order to deduce all angles as a function of time. Let's find $\theta(t)$.

i. Finding the pitch angle

We will use the concept of torque/moment balance to determine this angle. Intuitively, this means stating in an equations the condition for a system not to 'tip over'. Torque balance around the center of pressure yields:

$$I\ddot{\theta} + C\dot{\theta} = FLsin(\omega t)$$

Where I represents the *inertia of the system* and C is a *fluid damping coefficient* exerting a friction moment.

We can easily solve this analytically for $\theta(t)$, using appropriate boundary conditions. We get:

$$\theta(t) = \frac{FL}{C\omega} + \frac{FL}{I^2\omega^2 + C^2}\left(\frac{I^2\omega}{C} e^{\frac{-Ct}{I}} - \frac{C}{\omega}\cos(\omega t) - Isin(\omega t)\right)$$

ii. Finding the other angles

Now that we have $\theta(t)$, we will use relations (1) and (2) to determined the other angles. We can also plot these as functions of time:

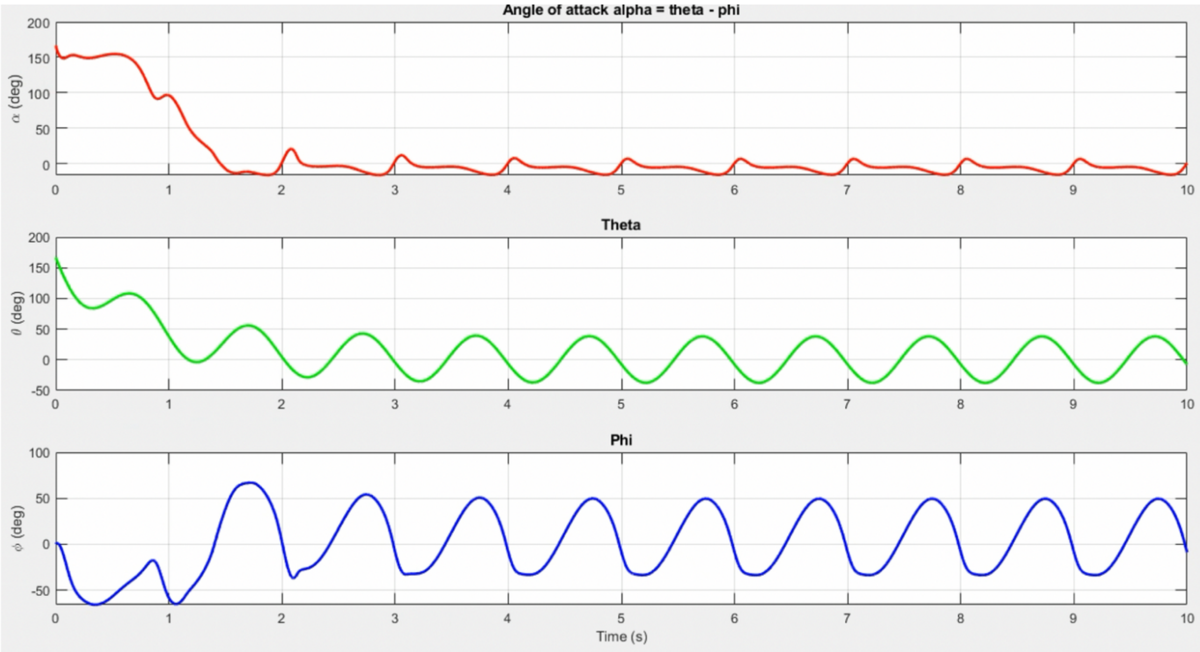

Figure 4: Temporal evolution of all three angles during the motion

d. Lift and drag coefficients

Now that we have the angles carefully plotted out, we can start determining the coefficients CL and CD. There is no mathematical for this, and my work was to determine accurate functions based on experimental values collected for our foil. The functions don't need to be exactly the same as the measured values, but it is essential they "behave" the same and have a similar order of magnitude.

Finding perfected functions demanded a lot of trial and error but in the end, careful associations of trigonometric hyperbolic functions (inspired by *J. Fluid Mech.* (2025), *vol.* 1014, A24 – Olivia Pomerenk & Leif Ristoph) gave accurate depictions of the coefficients:

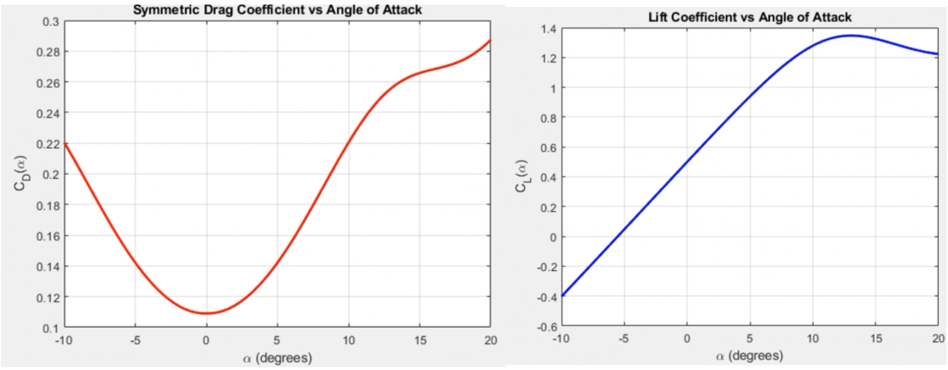

Figure 5: extrapolated function for the drag (left) and lift (right) coefficients

For reference, these are the experimental values which inspired the above functions:

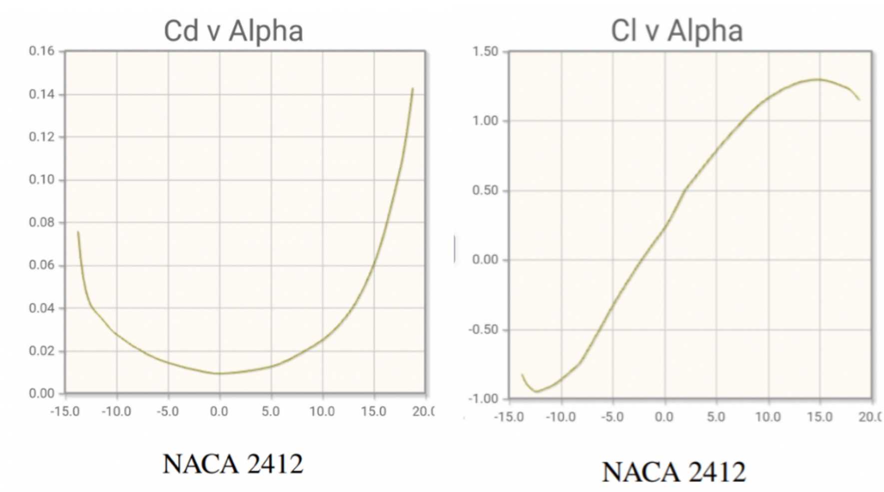

Figure 6: experimental values for the drag (left) and lift (right) coefficients

With all angles of motion determined and the aerodynamic coefficients explicitly stated, we can finally solve the differential equations.

3. Solving the equations

Recall our equations:

$$m\ddot{x} = -Lsin\phi - Dcos\phi$$

$$m\ddot{y} = Lcos\phi - Dsin\phi - Mg - Fsin(\omega t) + F_{arch}$$

Firstly, a bit of algebra is needed to rewrite these:

$$\ddot{x} = -\frac{1}{2m}A\rho\sqrt{\dot{x}^2 + \dot{y}^2}(C_L\dot{y} + C_D\dot{x})$$

$$\ddot{y} = -\frac{Mg}{m} - \frac{F}{m}\sin(\omega t) + \frac{F_{arch}}{m} + \frac{1}{2m}A\rho\sqrt{\dot{x}^2 + \dot{y}^2}(C_L\dot{x} - C_D\dot{y})$$

Using a MATLAB code, we can numerically solve this highly non-linear system of coupled differential equations. The result is the temporal evolution of the vertical position of the board around the waterline, and velocity with regards to both the x and y axis.

Using this model, work was done to test multiple sets of parameters (applied force, pumping frequency) , finding which allowed the best stability and highest x velocity. Eventually the optimal set was found to be:

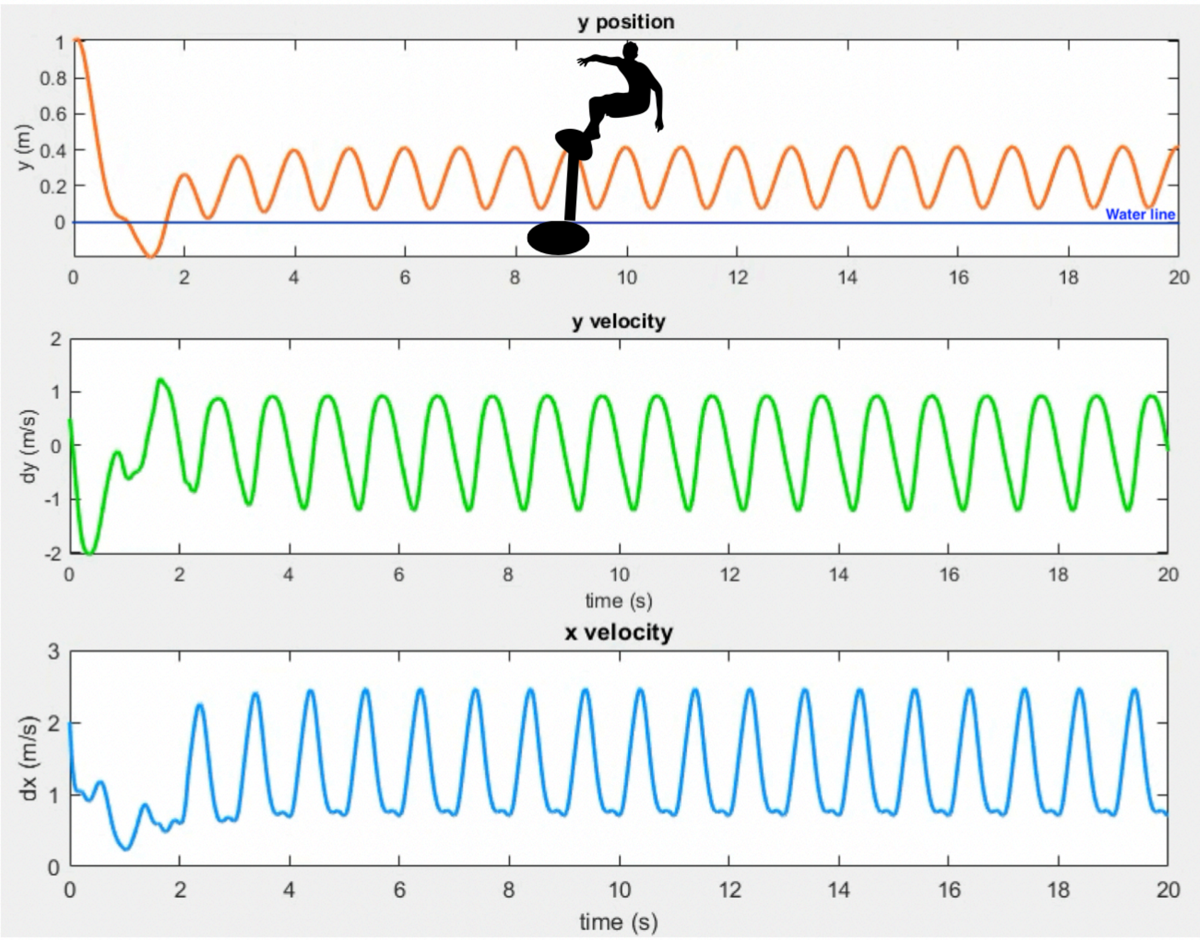

Figure 7: Results of numerical integration on MATLAB

$$\left(F_{opt}, f_{opt}\right) = (350\ N, 1\ Hz)$$

We can see above that with this model for an optimal set of parameters, the board remains at a steady altitude above the waterline, remaining stable. Also, we can see the x velocity ranges between around 1 and 2.5m/s, which is a steady advance rate.

a. Impact of applied force

- The minimum force needed for horizontal displacement is around 150N. Less force does not generate enough lift to propel the system forward.
- Above 500N, the board will sink as too much applied force will bring the system down. With this model, buoyancy prevented the system from sinking below the surface, but a board floating with no movement still is considered as a failure.
- The optimal applied strength found for a range of different pumping frequencies was 350N. This is the equivalent of lifting 30kg, and despite this seeming quite difficult to perform at a fast rhythm, keep in mind this force is applied by the foot, which is generally much better a distributing strength than per se, an arm.
- These results were done for an individual of 70kg. Logically, if the person weighs more, they have to push harder therefore applying more force.

b. Impact of pumping frequency

- The ‘best’ pumping frequency is hard to determine, for it depends on the applied force. In general, a higher force requires slower pumping and vice-versa.
- In general, however, this study has shown that for a realistic range of applied force, the optimal frequency was quite constant around 1Hz, so one ‘beat’ per second.
- Pumping too slow (Below 0.5Hz) results generally offers more vertical displacement, with the risk of the system reaching below the water lines. Also, the oscillations are much more irregular, showing the system is quite unstable in this case.
- In contrast, pumping too fast (above 3Hz) always keeps the board stable above the water line, but the system is actually slower in this case than for f=1Hz for example. Indeed, for high pumping frequencies the x velocity generally peaks at less than 1m/s which is far from optimal. This can be explained by the fact that when one pumps too fast, energy is disspated much faster within the fluid, slowing the system down.

These results highlight the trade-off between applied force and pumping frequency, showing that efficiency depends on maintaining a balance between lift generation and fluid dissipation. The identified optimal conditions align with biomechanical constraints, as a pumping frequency of around 1 Hz is feasible for human riders.

4. Conclusion

In conclusion, this work was successful in creating a working model for pump foil as well as determining the optimal set of parameters $(F_{opt}, f_{opt}) = (350\ N, 1\ Hz)$ allowing for most efficient transport on the water.

The system's stability was also understood through careful balance of forces and moments.

This model was designed as a foundation, which could be perfected for even more precise results during further study. Both labs will continue working on this project, implementing different ideas such as:

- Better understanding of the system's center of pressure
- Implementing the complex pitching moment into our torque balance (it was assumed to be zero as a first approximation)

5. Acknowledgements

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